EE 3140 HOMEWORK #1 SOLUTIONS

1.(a) The outcome of this experiment consists of a pair of numbers (x,y) where x= the number of dots in the 1st toss & y= the number of dots in the 2nd toss. Therefore, S=set of ordered pairs (x,y) where x, y ∈ {1,2,3,4,5,6} which are listed below:
(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(,5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)

(b) A= "Sum is even"
{(1,1),(1,3),(1,5),(2,2),(2,4),(2,6)(,3,1),(3,3),(3,5),(4,2)(4,4),(4,6),(5,1),(5,3),(5,5),(6,2),(6,4),(6,6)}

(c) B= "Both are even"
(2,2),(2,4)(2,6)(4,2),(4,4)(4,6),(6,2),(6,4),(6,6)

(d) B is a subset of A so when B occurs then A, thus B implies A.

(e) $A \cap B^c$ = "Sum is even and both tosses show an odd number"

{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5)}

(f) C = "Number of dots differ by 1"
{(1,2),(2,1),(2,3),(3,2)(3,4),(4,3),(4,5),(5,4),(5,6),(6,5)}
$A \cap C = \Phi$

2. (a) S={(1,2,3),(2,1,3),(3,1,2),(1,3,2),(2,3,1),(3,2,1)}

(b) A_1 = {(1,2,3),(1,3,2)}
A_2 = {(1,2,3),(3,2,1)}
A_4 = {(1,2,3),(2,1,3)}

(c) $A_1 \cap A_2 \cap A_3$ = {(1,2,3)}
"The number of every ball corresponds to the number of the draw"

(d) $A1 \cup A2 \cup A_3$ = {(1,2,3),(1,3,2),(3,2,1),(2,1,3)}
"The number of atleast one ball corresponds to the number of the draw."

(e) $(A_1 \cup A_2 \cup A_3)^c$ = {(3,1,2),(2,3,1)}
"None of the balls has a number that corresponds to the number of the draw"

3.(a) $P(A_2) = P[\{(1,1)\}] = 1/36$

$P(A_3) = P[\{(1,2),(2,1)\}] = 2/36$

$P(A_4) = P[\{(2,2),(1,3),(3,1)\}] = 3/36$

$P(A_5) = P[\{(1,4),(2,3),(3,2),(4,1)\}] = 4/36$

$P(A_6) = P[\{(1,5),(2,4),(3,3),(4,2),(5,1)\}] = 5/36$

$P(A_7) = P[\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}]= 6/36$

$P(A_8) = P[\{(2,6),(3,5),(4,4),(5,3),(6,2)\}] = 5/36$

$P(A_9) = P[\{(3,6),(4,5),(5,4),(6,3)\}] = 4/36$

$P(A_{10}) = P[\{(4,6),(5,5),(6,4)\}] = 3/36$

$P(A_{11}) = P[\{(5,6),(6,5)\}] = 2/36$

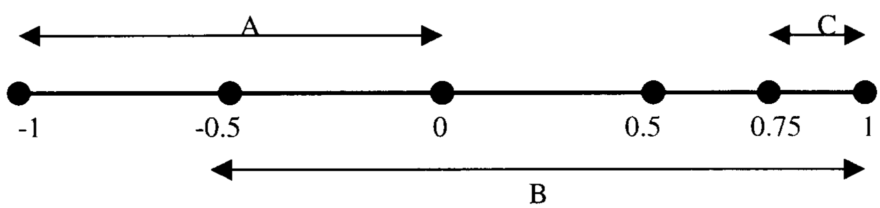
$P(A_{12}) = P[\{6,6)\}] = 1/36$

(b) Let B = {outcomes of two tosses are the same}. Then

$P[B] = P[\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}] = 6/36 = 1/6$

P[outcomes of two tosses are different] = $P[B^c] = 1-1/6 = 5/6$

4.(a)



From the figure $P(B) = ¾$; $P(A\cap B) = ¼$; $P(A\cap C) = 0$

$P(A\cup B) = P(A) + P(B) - P(A\cap B) = ½ + ¾ - ¼ = 1$

$P(AUC) = P(A) + P(C) - P(A\cap C) = ½ + {}^1/_8 - 0 = {}^5/_8$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cup B) - P(B\cup C) - P(C\cup A) + P(A\cap B\cap C)$

$= ½ + ¾ + 1/8 - ¼ - 1/8 - 0 + 0 = 1$

5. 6.2.52 = 624

6. On the first day the student can wear any of the four pairs; on any subsequent day he has a choice from 3 pairs, therefore:

The total # of distinct ordered 5-tuples = 4.3.3.3.3 = 324

7. $9!/(4!2!3!) = 1260$

8. A: total # of dots even ; B: both tosses even

Here, $A\cap B = B$ as $B\subset A$ $\therefore P(A\cap B) = P(B)$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

$$P(B|A) = \frac{P(A\cap B)}{P(B)} = \frac{P(B)}{P(A)} = \frac{1}{2}$$

9. P(2 or more students have the same birthday)
= 1- P(no 2 students have the same birthday)

$$= 1 - \frac{365.364\ldots\ldots(365\text{-}20\text{+}1)}{365^{20}} = 0.412$$

10. Let P(chip is defective) = P(def)
Given, P(def | A) = 0.001 ; P(def | B) = 0.005 ; P(def | C) = 0.01

Using Bayes' Theorem,

$$P(A \mid def) = \frac{P(def \mid A)\,P(A)}{P(def \mid A)P(A) + P(def \mid B)P(B) + P(def \mid C)P(C)}$$

$$= \frac{.001P(A)}{.001P(A) + .005P(B) + .01P(C)}$$

$$= \frac{P(A)}{P(A) + 5P(B) + 10P(C)}$$

Similarly,

$$P(C \mid def) = \frac{10P(C)}{P(A) + 5P(B) + 10P(C)}$$

11. $P(A) = P(A\cap B) + P(A\cap B^C)$ by corollary 1

$P(A\cap B^C) = P(A) - P(A\cap B)$
$= P(A) - P(A)P(B)$ [since A & B are independent]
$= P(A)[1 - P(B)]$
$= P(A)P(B^c)$ $\therefore$ A & B^c are independent.

Similarly ,
$P(B) = P(A\cap B) + P(A^c\cap B) = P(A)P(B) + P(A^c\cap B)$
$\Rightarrow P(A^c\cap B)= P(B)[1 - P(A)]$
$= P(B)P(A^c)$ $\therefore$ B & A^c are independent.

Finally,
$P(A^c) = P(A^c\cap B) + P(A^c\cap B^c) = P(A^c)P(B) + P(A^c\cap B^c)$
$\Rightarrow P(A^c\cap B^c) = P(A^c)[1 - P(B)] = P(A^c)P(B^c)$ $\therefore A^c$ & B^c are independent

12. (a) $P(A\cup B) = P(A) + P(B) - P(A\cap B) = P(A) + P(B) - P(A)P(B)$
(b) $P(A\cup B) = P(A) + P(B) - P(A\cap B) = P(A) + P(B)$

13. P(3 or more errors)= 1-P(2 or fewer errors)

$$1 - \sum_{k=0}^{100} \binom{100}{k} (.001)^k (.999)^{100-k}$$

$= 1 - [(.999)^{100} + 100(.001)(.999)^{99} + \frac{100.99(.999)^{98}}{2}(.001) = 1.5 \times 10^{-4}$

14. Probability of success = 95/100 = p
Choose *n* such that $P(k \geq 10) \geq 0.9$
Where

$$P(k\geq 10) = \binom{n}{k} p^k(1-p)^{n-k} = {}^nC_k\ p^k(1-p)^{n-k}$$

for n = 11 $P(k\geq 10) = .8981$
for n = 12 $P(k\geq 10) = .98$ $\therefore$ pick n=12

15. (a) $P(\text{k erors}) = \binom{n}{k} p^k(1-p)^{n-k}$

(b) Probability of type 1 errors = ap ;

$$P(k_1 \text{ type 1 errors}) = {}^nC_{k1}(pa)^{k1}(1\text{-}pa)^{n-k1}$$

(c) Probability of type 2 errors = (1-a)p

$$P(k_2 \text{ type 2 errors} = {}^nC_{k2}\,[p(1\text{-}a)]^{k2}\,[1 - p(1\text{-}a)]^{n-k2}$$

(d) Total outcomes =3 ie. Type 1 error, type 2 error and no error

$$P(k_1, k_2, n - k_1 - k_2) = \frac{n!}{k_1!\ k_2!\ (n - k_1 - k_2)!}(pa)^{k1}[p(1\text{-}a)]^{k2}\,[1 - p]^{n-k1-k2}$$

16. P(H) = 0.1 ; P(N) = 0.4 ; P(L) = 0.5

(a) Binomial probability: ${}^nC_k\ (0.9)^k\ (0.1)^{n-k}$

(b) Geometric Probability: $(0.9)^k . (0.1)$

(c) Multinomial Probability: $P(5,10,5) = \frac{20!}{5!\ 10!\ 5!}(0.1)^5(0.4)^{10}(0.5)^5$

17. (a) A car pays k dollars if the parking time is in the range ((k-1)/2 , k/2]

$$P(\text{paying k dollars}) = P[\,(k\text{-}1)/2 < t \leq k/2\,]$$
$$= P[\,t > (k-1)/2] - P[t > k/2\,]$$

$$= e^{-(k\text{-}2)/2} - e^{-k/2} \quad k= 1,2....$$

(b) For $1 \leq k \leq 4$, P(k) same as in part (a)

$P(k=5) = P[t> (5-1)/2] = e^{-2}$

18. (a) The # of black balls in the urn completely determines the probability of the outcomes of the next experiment.

$P(www) = (1/2)(1/2)(1/2) = 1/8$
$P(bww) = (1/2)(2/3)(2/3) = 2/9$
$P(bbw) = (1/2)(1/3)(2/2) = 1/6$
$P(bbwww) = (1/2)(1/3).1.1.1 = 1/6$

(b) Both black balls should be removed in *3* trials
$P(bbw) + P(bwb) + P(wbb) = (1/2)(1/3).1 + (1/2)(2/3)(1/3 + (1/2)(1/2)(1/3) = 13/36$

(c) $P(\underbrace{www......w}_{n\text{ times}}) = (1/2)^n$

19. $Y_n = (b-a)\,U_n + a$, $\therefore \alpha = b - a$; $\beta = a$

20. (a) P(ace in the first draw) = 4/52 = 1/13

(b) Suppose the first draw is seen to be an ace; then
P(ace in 2nd draw | ace in the first draw) = 3/51
Suppose the first draw was observed not as an ace; then
P(ace in the 2nd draw | no ace in the 1st draw) = 4/51

If we do not look at the 1st draw then
P(ace in 2nd draw) = P(ace in 2nd draw | ace in 1st draw)P(ace in 1st draw)
+ P(ace in 2nd draw | no ace in 1st draw)P(no ace in 1st draw)
= (3/51)(4/52) + (4/51) (48/52) = 4/52 = P(ace in 1st draw)

Therefore, the changes on observing the first draw.

(c) $P(\text{3 aces in 7 cards}) = \dfrac{{}^4C_3 \cdot {}^{48}C_4}{{}^{52}C_7} = 0.00582$ ......(1)

$P(\text{2 kings in 7 cards}) = \dfrac{{}^4C_2 \cdot {}^{48}C_5}{{}^{52}C_7} = 0.0768$ ......(2)

$P(\text{3 aces } \textit{and} \text{ 2 kings}) = \dfrac{{}^4C_3\,{}^4C_2\,{}^{44}C_2}{{}^{52}C_7} = 1.697 \times 10^{-4}$ .....(3)

(Note: (1) & (2) are not independent)

$P(\text{3 aces } \textit{or} \text{ 2 kings}) = \dfrac{{}^4C_3 \cdot {}^{48}C_4}{{}^{52}C_7} + \dfrac{{}^4C_2 \cdot {}^{48}C_5}{{}^{52}C_7} - \dfrac{{}^4C_3\,{}^4C_2\,{}^{44}C_2}{{}^{52}C_7} = 0.08245$

(d) $\dfrac{{}^4C_1\,{}^{48}C_{12}\,{}^3C_1\,{}^{36}C_{12}\,{}^2C_1\,{}^{24}C_{12}\,{}^1C_1\,{}^{12}C_{12}}{{}^{52}C_{13}\,{}^{39}C_{13}\,{}^{26}C_{13}\,{}^{13}C_{13}} = 0.105498$