EE 3140 Probability and Random Process Homework2#

1. An urn contains 90 \$1 bills, 9 \$5 bills, and 1 \$50 bill. Let the random variable X be the denomination of a bill that is selected at random from the urn.
   a. Describe the underlying space S of this random experiment and specify the probabilities of its elementary events.
   b. Describe the sample space of X, S_X, and find the probabilities for the various values of X.

2. An information source produces symbols at random from a five-letter alphabet: S = {a, b, c, d, e}. The probabilities of the symbols are:

   P(a)= ½, p(b)= ¼, p(c)= 1/8, and p(d)= p(e)= 1/16.

   A data compression system encodes the letters into binary strings as follows:

A	1
B	01
C	001
D	0001
E	0000

   Let the random variable Y be equal to the length of the binary string output by the system. Specify the sample space of Y, S_Y, and the probabilities of its values.

3. Let U be a uniform random variable in the interval [-1,1]. Find the following probabilities:

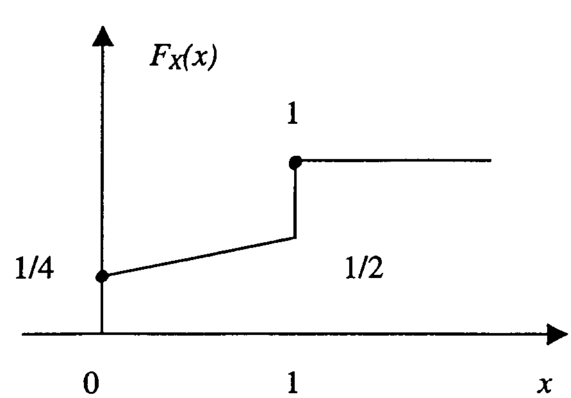
   $P[U > 0]$ $P[|U| < 1/3]$ $P[|U| >= 3/4]$
   $P[U < 5]$ $P[1/3 < U < 1/2]$

4. The cdf of the random variable X is given by

$$F_X(x) = \begin{cases} 1/3 + (2/3)(x+1)^2, & -1 \le x \le 0 \\ 0 & x < -1 \end{cases}$$

   Find the probability of the event $A = \{X > 1/3\}$, $B = \{|X| >= 1\}$, $C = \{|X - 1/3| < 1\}$, $D = \{X < 0\}$.

5. The cdf of a random variable X is shown in Fig. P3.1.



a. What type of random variable is X?
b. Find the following probabilities in term of the cdf of X:

P[X < -1/2]	P[X < 0]	P[X <= 0]
P[1/4 <= X < 1]	P[1/4 <= X <= 1]	P[X > 1/2]
P[X >= 5]	P[X <5].	

6. A continuous random variable X has cdf

$$F_X(x) = \begin{cases} 0 & x \le -\pi/2 \\ c(1+\sin(x)), & -\pi/2 \le x \le \pi/2 \\ 1 & \pi/2 \le x \end{cases}$$

a. Find c.
b. Plot $F_X(x)$.

7. The Rayleigh random variable has cdf

$$F_R(r) = \begin{cases} 0, & r < 0 \\ 1 - e^{-r^2/2\sigma^2}, & r \ge 0 \end{cases}$$

Find P[σ <= R <= 2σ] and P[R> 3σ].

8. A random variable X has pdf

$$f_X(x) = \begin{cases} cx(1-x), & -1 \le x \le 0 \\ 0 & elsewhere \end{cases}$$

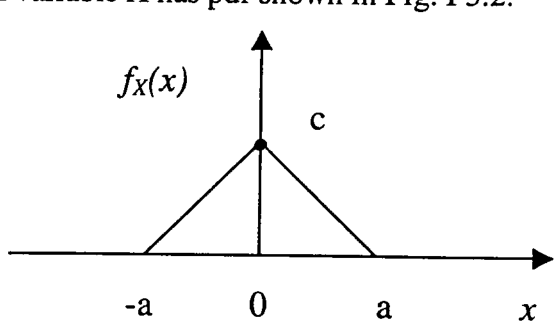
a. Find c.
b. Find P[1/2 <= X < 3/4].
c. Find $F_X(x)$.

9. A random variable X has pdf

$$f_X(x) = \begin{cases} c(1-x^4), & -1 \le x \le 1 \\ 0 & elsewhere \end{cases}$$

a. Find c.
b. Find the cdf of X.
c. Find $P[|X| < 1/2]$.

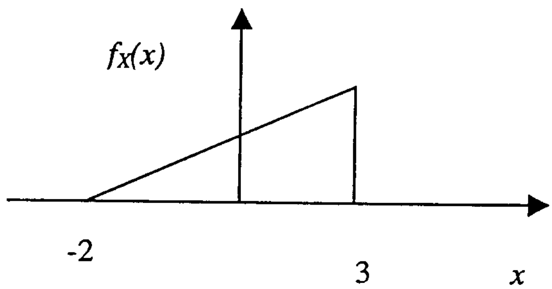
10. A random variable X has pdf shown in Fig. P3.2.



a. Find $f_X(x)$.
b. Find the cdf of X.
c. Find b such that $P[|X| < b] = ½$.

11. Let the random variable Y be defined by $Y = X^2$, where X is a continuous random variable. Find the cdf and pdf of Y.

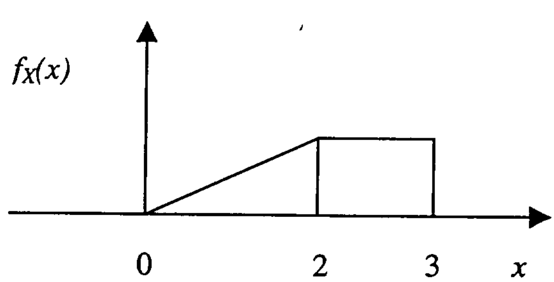
12. The pdf of X is shown as the following Figure. Find and sketch the pdf f(y) of $Y = X^2$



13. Find and sketch the pdf [$f(x)$] if the cdf is as given below:

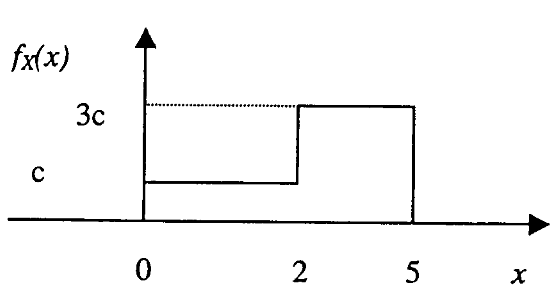
$$F(x) = \begin{cases} e^x/2, & x \le 0 \\ 1/2 & 0 \le x \le 2 \\ (1+x)/6 & 2 \le x \le 5 \\ 1 & x \ge 5 \end{cases}$$

14. For the random variable with the pdf sketched below,



Find E[X] and VAR[X].

15. Find the variance of the random variable sketched below:



16. A random number generator produces the data: 2, 1, 1, 3, 4, 2, 0, 1, 2, 0, 7, 1
Find the variance of this data.

17. Find the mean and variance of the random variable X that is uniformly distributed in the interval [a, b].

18. Find the mean and variance of the Geometric random variable.

19. Find the mean and variance of the Binomial random variable.

20. Find the mean and variance of the Poisson random variable.