

Homework 1**Assignment for Course: EE 3140 Probability and Random Process**

1. A die is tossed twice and the number of dots facing up is counted and noted in the order of occurrence.
 - a. Find the sample space.
 - b. Find the set A corresponding to the event "total number of dots showing is even."
 - c. Find the set B corresponding to the event "both dice are even."
 - d. Does A imply B or B imply A ?
 - e. Find $A \cap B^c$ and describe this event in words.
 - f. Let C correspond to the event "number of dots in die differ by 1." Find $A \cap C$.
2. The three balls numbered 1 to 3 in an urn are drawn at random one at a time until the urn is empty. The sequence of the ball number is noted.
 - a. Find the sample space.
 - b. Find the sets A_k corresponding to the events "ball number k is selected in the k th draw," for $k=1, 2, 3$.
 - c. Find the set $A_1 \cap A_2 \cap A_3$ and describe the event in words.
 - d. Find the set $A_1 \cup A_2 \cup A_3$ and describe the event in words.
 - e. Find the set $(A_1 \cup A_2 \cup A_3)^c$ and describe the event in words.
3. A die is tossed twice and the number of dots facing up noted in the order of occurrence. Assuming that all outcomes are equally likely to occur, find the probabilities of the following events:
 - a. A_k : the sum of the two outcomes is k , for $k=2, 3, \dots, 12$.
 - b. B : the outcomes of the two tosses are different.
4. A number x is selected at random in the interval $[-1, 1]$. Let the events $A = \{x < 0\}$, $B = \{|x - 0.5| < 1\}$, and $C = \{x > 0.75\}$.
 - a. Find the probabilities of B , $A \cap B$, and $A \cap C$.
 - b. Find the probabilities of $A \cup B$, $A \cup C$, and $A \cup B \cup C$, first, by directly evaluating the sets and then their probabilities, and second, by using the appropriate axioms or corollaries.
5. A six-sided die is tossed, a coin is flipped, and a card is selected at random from a deck of 52 distinct cards. Find the number of possible outcomes.
6. A student has four different pairs of shoes and never wears the same pair on two consecutive days. In how many ways can he wear shoes in 5 days?
7. How many distinct permutation are there of four red balls, two white balls, and three black balls.

8. A die is tossed twice and the number of dots facing up is counted and noted in the order of occurrence. Let A be the event "total number of dots is even," and let B be the event "both tosses had an even number of dots." Find $P[A | B]$ and $P[B | A]$.
9. Find the probability that 2 or more students in a class of 20 students have the same birthday.
10. A computer manufacturer uses chips from three sources. Chips from sources A , B , and C are defective with probabilities .001, .005 and .01, respectively. If a randomly selected chip is found to be defective, find the probability that the manufacturer was A ; that the manufacturer was C .
11. Show that if A and B are independent events, then the pairs A and B^c , A^c and B , and A^c and B^c are also independent.
12. Let A and B be events with probabilities $P[A]$ and $P[B]$.
 - a. Find $P[A \cup B]$ if A and B are independent.
 - b. Find $P[A \cup B]$ if A and B are mutually exclusive.
13. A block of 100 bits is transmitted over a binary communication channel with probability of bit error $p = 10^{-3}$. Find the probability that the block contains three or more errors.
14. A student needs 10 chips of a certain type to build a circuit. It is known that 5% of these chips are defective. How many chips should he buy for there to be a greater than 90% probability of having enough chips for a circuit?
15. A machine makes errors in a certain operation with probability p . There are two types of errors. The fraction of errors that are type 1 is a , and the fraction that are type 2 is $1 - a$.
 - a. What is the probability of k errors in n operations?
 - b. What is the probability of k_1 type 1 errors in n operations?
 - c. What is the probability of k_2 type 2 errors in n operations?
 - d. What is the joint probability of k_1 and k_2 type 1 and 2 errors, respectively, in n operations?
16. Three types of messages arrive at a message center. Ten percent of the messages are "high priority," 40 percent are "normal priority," and 50 percent are "low priority."
 - a. Find the probability that k out of N messages are not high priority.
 - b. Suppose that messages arrive one at a time. Find the probability that k messages are received before a high-priority message arrives.
 - c. Find the probability that out of 20 messages, 5 are high priority, 10 are normal priority, and 5 are low priority.

17. The amount of time cars are parked in a parking lot follows an exponential probability law with parameter 1. The charge for parking in the lot is \$1 for each half-hour or less.
- Find the probability that a car pays k dollars.
 - Suppose that there is a maximum charge of \$5. Find the probability that a car pays k dollars.
18. An urn initially contains two black balls and two white balls. The following experiment is repeated indefinitely: A ball is drawn from the urn; if the ball is white it is put back in the urn, otherwise it is left out.
- Find the probabilities for the sequences www , bww , bbw , and $bbwww$.
 - Find the probability that the urn contains no black balls after three draws.
 - Find the probability that the urn contains two black balls after n trials.
19. Suppose you have a program that gives you numbers U_n that are uniformly distributed in the interval $[0, 1]$. Let $Y_n = \alpha U_n + \beta$. Find α and β if Y_n is uniformly distributed in the interval $[a, b]$.
20. Consider a well-shuffled deck of cards consisting of 52 distinct cards, of which four are aces and four are kings.
- Find the probability of obtaining an ace in the first draw.
 - Draw a card from the deck and look at it. What is the probability of obtaining an ace in the second draw? Does the answer change if you had not observed the first draw?
 - Suppose we draw 7 cards from the deck. What is the probability that the 7 cards include 3 aces? What is the probability that 7 cards include 2 kings? What is the probability that the 7 cards include 3 aces and/or 2 kings?
 - Suppose that the entire deck of cards is distributed equally among four players. What is the probability that each player gets an ace?