EE 3140 Homework #3

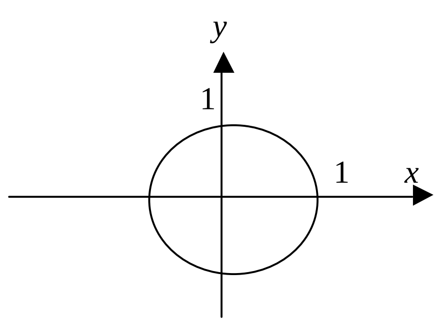
1. Let the random variables X, Y, and Z be independent random variables. Find the following probabilities in terms of $F_X(x)$, $F_Y(y)$, and $F_Z(z)$.
   a. $P[X < 6, |Y| < 2, Z^2 > 2]$
   b. $P[X < 5, Y = 0, Z \leq 1]$
   c. $P[\min(X, Y, Z) > 0]$
2. A die is tossed twice; let X_1 and X_2 denote the outcome of the first and second toss, respectively.
   a. What is the joint pmf for (X_1, X_2) if the tosses are independent and if the outcomes of each toss are equiprobable?
   b. Let $X = \min(X_1, X_2)$ and $Y = \max(X_1, X_2)$. Find the joint pmf for (X_1, X_2).
   c. Find the marginal pmf's for X and Y in part b.
3.
   a. Find the marginal pmf's for the pairs of random variables with the indicated joint pmf.

Y \ X	-1	0	1
-1	1/6	0	1/6
0	0	0	1/3
1	1/6	0	1/6

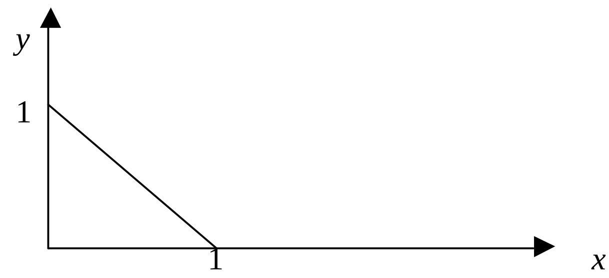
Y \ X	-1	0	1
-1	0	1/8	1/8
0	1/2	0	0
1	0	1/8	1/8

Y \ X	-1	0	1
-1	0	0	0
0	0	1/3	0
1	1/3	0	1/3

   b. Find the probability of the events A = $\{X \leq 0\}$, B = $\{X \leq Y\}$, and C = $\{X = -Y\}$ for the above joint pmf's
4. The random vector (X, Y) has the joint pdf
$$f(x,y) = 2e^{-x}e^{-2y} \qquad x > 0,\ y > 0$$
   a. Find the joint cdf.
   b. Find $P[X > Y]$
   c. Find the marginal pdf's
5. The random vector variable (X, Y) has the joint pdf
$$f(x,y) = k(x+y) \qquad 0 < x < 2,\ 0 < y < 2.$$
   a. Find k.
   b. Find the joint cdf of (X, Y).
   c. Find the marginal pdf of X and Y.
6. The random vector (X, Y) is uniformly distributed inside the regions shown in the figure below, and zero elsewhere. Find the value of k and the marginal pdf's.



7. The random vector (X, Y) is uniformly distributed inside the regions shown in the figure below, and zero elsewhere. Find the value of k and the marginal pdf's.



8. The random vector (X, Y) has a joint pdf

$$f_{X,Y}(x,y) = 2e^{-x}e^{-2y} \qquad x > 0, y > 0$$

Find the probability of the following events:
   a. $\{X + Y \leq 5\}$
   b. $\{X < Y\}$
   c. $\{X - Y \leq 8\}$
   d. $\{X^2 < Y\}$
9. Let (X,Y) have the joint pdf

$$f_{X,Y}(x,y) = xe^{-x(1+y)} \qquad x > 0, y > 0$$

Find the marginal pdf of X and of Y.
10. Are X and Y independent random variables in Problem 3?
11. Are X and Y independent random variables in Problem 4?
12. Are X and Y independent random variables in Problem 5?
13. Consider a sequence of $n + m$ independent Bernoulli trials with probability of success p in each trial. Let N be the number of successes in the first n trials and let M be the number of successes in the remaining m trials.
   a. Why are N and M independent random variables?
   b. Find the joint pmf of N and M and the marginal pmf's of N and M.
   c. Find the pmf for the total number of successes in the $n + m$ trials.
14. Find the conditional pmf's of Y given X = -1 for all three cases in Problem 3.
15. Find $f_Y(y \mid x)$ in Problem 5.
16. Let (X, Y, Z) have joint pdf

$$f_{X,Y,Z}(x, y, z) = k(x + y + z) \quad 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$$

   a. Find k
   b. Find $f_Z(z \mid x, y)$.
17. Show that $f_{X,Y}(x, y, z) = f_Z(z \mid x, y) f_Y(y \mid x) f_X(x)$.
18.
   a. Find $E[(X + Y)^2]$
   b. Find the variance of $X + Y$.
   c. Under what condition is the variance of the sum equal to the sum of the individual variances?
19. Find $E[| X - Y|]$ if X and Y are independent exponential random variables with parameter α =1.
20. Find $E[X^2Y]$ where X is a zero-mean, unit-variance Gaussian random variable, and Y is a uniform random variable in the interval [-1,3], and X and Y are independent.