## EE 3140 Homework 2

1. Suppose that a point is selected at random from inside the unit circle. Let $Y$ be the distance of the point from the origin.
a. Find the sample space of $Y, S_{Y}$
b. Find the equivalent event in $S$ for the event $\{Y \leq y\}$
c. Find $P[Y \leq y]$
2. Plot the cdf of the radius $Y$ in Problem 1. Specify the type of $Y$.
3. The cdf of the random variable $X$ is given by

$$
\begin{aligned}
F_{X}(x) & =1 / 3+(2 / 3)(x+1)^{2} & & -1 \leq x \leq 0 \\
& =0 & & x<-1
\end{aligned}
$$

Find the probability of the events $A=\{X>1 / 3\}, B=\{|X| \geq 1\}, C=\{|X-1 / 3|<1\}, D=$ $\{X<0\}$
4. The cdf of a random variable $X$ is shown below
a. What type of random variable is $X$
b. Find the following probabilities in terms of the cdf of $X$

$$
\begin{array}{lll}
P[X<-1 / 2] & P[X<0] & P[X \leq 0] \\
P[1 / 4 \leq X<1] & P[1 / 4 \leq X \leq 1] & P[X>1 / 2] \\
P[X \geq 5] & P[X<5] &
\end{array}
$$


5. A random variable $Y$ has the cdf

$$
\begin{aligned}
F_{Y}(y) & =0 & & y<1 \\
& =1-y^{-n} & & y \geq 1
\end{aligned}
$$

where n is a positive integer.
a. Plot the cdf of $Y$.
b. Find the probability $P[k<Y \leq k+1]$ for a positive integer $k$.
6. A continuous random variable $X$ has cdf

$$
\begin{aligned}
F_{X}(x) & =0 \\
& =c[1+\cos (x)] \\
& =1
\end{aligned}
$$

a. Find $c$.
b. Plot $F_{X}(x)$
7. A random variable $X$ has pdf
$f_{X}(x)=c x(1-x) \quad 0 \leq x \leq 1$
a. Find $c$.
b. Find $P[3 / 4 \leq X \leq 1]$
c. Find $F_{X}(x)$
8. A random variable $X$ has pdf

$$
\begin{aligned}
f_{X}(x) & =c\left(1-x^{4}\right) & & -1 \leq x \leq 1 \\
& =0 & & \text { elsewhere }
\end{aligned}
$$

a. Find $c$
b. Find the cdf of $X$
c. Find $P[|X|<1 / 2]$
9. A random variable X has pdf shown below
a. Find $f_{X}(x)$.
b. Find the cdf of $X$
c. Find b such that $P[|X|<\mathrm{b}]=3 / 4$

10. Messages arrive at a computer at an average rate of 15 messages per second. The number of messages that arrive in 1 second is known to be a Poisson random variable.
a. Find the probability that no messages arrive in 1 second
b. Find the probability that more than 10 messages arrive in a 1 -second period.

Hint: Use Eq. (3.33a)
11. The pdf of $X$ is as shown below. Find the pdf of $Y$ where $Y=X^{2}$.

12. For the random variable $X$ whose pdf is shown below find the pdf of $Y$, if $Y=X^{3}$

13. Find the pdf for the cdf given below.

$$
\begin{array}{rlrl}
F_{X}(x) & =x^{2} / 2 & & 0 \leq x \leq 1 \\
& =1 / 2 & 1<x \leq 2 \\
& =1 / 2(x-1) & & 2<x \leq 3 \\
& =1 & & 3 \leq x
\end{array}
$$

14. Find $\operatorname{Var}[X]$ for the random variable shown below.

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16. Find the variance of the data
$3,2,2,4,5,3,1,0,3,1,6,1$
17. Find the mean and variance of a random variable uniformly distributed in the interval [a,b]
18. Find the mean and variance of a discrete random variable that takes on the values for the set $\{1,2, \ldots, n\}$ with equal probability.
19. Derive the mean and variance of the binomial random variable.
20. Derive the mean and variance of the poisson random variable.
