## EE 3140 Homework 2

- 1. Suppose that a point is selected at random from inside the unit circle. Let *Y* be the distance of the point from the origin.
  - a. Find the sample space of Y,  $S_Y$
  - b. Find the equivalent event in *S* for the event {  $Y \le y$  }
  - c. Find  $P[Y \le y]$
- 2. Plot the cdf of the radius *Y* in Problem 1. Specify the type of *Y*.
- 3. The cdf of the random variable *X* is given by

Find the probability of the events  $A = \{X > 1/3\}, B = \{|X| \ge 1\}, C = \{|X - 1/3| < 1\}, D = \{X < 0\}$ 

- 4. The cdf of a random variable *X* is shown below
  - a. What type of random variable is *X*
  - b. Find the following probabilities in terms of the cdf of X

$$\begin{array}{ll} P[X < -1/2] & P[X < 0] & P[X \le 0] \\ P[1/4 \le X < 1] & P[1/4 \le X \le 1] & P[X > 1/2] \\ P[X \ge 5] & P[X < 5] \end{array}$$



5. A random variable *Y* has the cdf

$$F_{Y}(y) = 0$$
  $y < 1$   
= 1 -  $y^{-n}$   $y \ge 1$ 

where n is a positive integer.

- a. Plot the cdf of *Y*.
- b. Find the probability  $P[k < Y \le k + 1]$  for a positive integer *k*.
- 6. A continuous random variable X has cdf

$$F_X(x) = 0 x \le -\pi/2 = c[1 + \cos(x)] -\pi/2 < x \le 0 = 1 0 < x$$

a. Find *c*.

b. Plot  $F_X(x)$ 

7. A random variable *X* has pdf

$f_X(x) = cx(1-x)$	$0 \le x \le 1$
= 0	elsewhere
a Find c	

- a. Find c.
- b. Find  $P[3/4 \le X \le 1]$

c. Find  $F_X(x)$ 

8. A random variable *X* has pdf

 $f_X(x) = c(1 - x^4) \qquad -1 \le x \le 1$ = 0 elsewhere

a. Find 
$$\alpha$$

b. Find the cdf of 
$$\lambda$$

- c. Find P[|X| < 1/2]
- 9. A random variable X has pdf shown below
  - a. Find  $f_X(x)$ .
  - b. Find the cdf of X
  - c. Find b such that P[|X| < b] = 3/4



- 10. Messages arrive at a computer at an average rate of 15 messages per second. The number of messages that arrive in 1 second is known to be a Poisson random variable.
  - a. Find the probability that no messages arrive in 1 second
  - b. Find the probability that more than 10 messages arrive in a 1-second period. *Hint:* Use Eq. (3.33a)
- 11. The pdf of X is as shown below. Find the pdf of Y where  $Y = X^2$ .



12. For the random variable X whose pdf is shown below find the pdf of Y, if  $Y = X^3$ 



13. Find the pdf for the cdf given below.

$F_X(x)$	$= x^{2}/2$	$0 \le x \le 1$
	= 1/2	$1 < x \leq 2$
	= 1/2(x - 1)	$2 < x \leq 3$
	= 1	$3 \le x$

14. Find Var[X] for the random variable shown below.



15. Find Var[X] for the random variable shown below.



- 16. Find the variance of the data 3, 2, 2, 4, 5, 3, 1, 0, 3, 1, 6, 1
- 17. Find the mean and variance of a random variable uniformly distributed in the interval [a,b]
- 18. Find the mean and variance of a discrete random variable that takes on the values for the set  $\{1, 2, ..., n\}$  with equal probability.
- 19. Derive the mean and variance of the binomial random variable.
- 20. Derive the mean and variance of the poisson random variable.