

EE 3140 Homework #1

1. A die is tossed and the number of dots facing up is counted and noted.
 - a. What is the sample space?
 - b. What is the set A corresponding to the event even number of dots are facing up ?
 - c. Find the set A^c and describe the corresponding even in words.
2. A random experiment has sample space $S = \{a, b, c\}$. Suppose that $P[\{a, c\}] = 5/8$ and $P[\{b, c\}] = 7/8$. Use the axioms of probability to find the probabilities of the elementary events $P[\{a\}]$, $P[\{b\}]$ and $P[\{c\}]$.
3. How many computer passwords containing eight distinct (no letter repeated) letters from the 26 of the alphabet can be constructed?
4. In how many ways can a standard deck of 52 playing cards be dealt into 4 hands of 13 cards each?
5. A fair coin is tossed four times. Let A_i be the event i th toss results in heads. Find the probabilities of the following events: A_2 , $A_1 \cap A_3$, $A_1 \cap A_2 \cap A_3 \cap A_4$, and $A_1 \cup A_2 \cup A_3 \cup A_4$.
6. 30 defective diodes have been mixed with 200 non-defective ones. What is the probability that if 20 diodes are picked at random, exactly k will be defective? ($k \leq 20$)
7. An urn contains six red, four white, and eight blue balls. Consider the following situations:
 - a. Five balls are drawn with replacement. What is the probability of obtaining three red?
 - b. Five balls are drawn without replacement. What is the probability of obtaining three red?
 - c. Five balls are drawn with replacement. What is the probability of obtaining two red, two white, and one blue?
 - d. Five balls are drawn without replacement. What is the probability of obtaining two red, two white and one blue?
8. An engineer has determined that 80% of all motor failures are due to either leaking joints or bearing overheating and that these two occur with equal probability. The remaining failures are due to miscellaneous causes. Among ten failures, what is the probability that six of them were due to bearing overheating, two due to leaking joints, and two due to other causes?
9. How many seven-digit telephone numbers are possible if the first number is not allowed to be 0 or 1?
10. In how many ways can 10 students occupy 10 desks? 12 desks?
11. Find the probability that the sum of the outcomes of three tosses of a die is 7.
12. Show that $P[A \cap B \cap C] = P[A | B \cap C]P[B | C]P[C]$.
13. One of two coins is selected at random and tossed. The first coin comes up heads with probability p_1 and the second coin with probability p_2 .
 - a. What is the probability that the outcome of the toss is heads?
 - b. What is the probability that coin 2 was used given that a heads occurred?

14. A survey shows that 30% of men and 25% of women smoke cigarettes, and among men 40% prefer the Marlboro brand, and among women 70% prefer the Marlboro brand. If a person is found smoking a Marlboro cigarette, what is the probability that she is a woman?
15. Three boxes each contain two coins. In one box, B_1 , both coins are gold, in another, B_2 , both are silver, and in the third, B_3 , one is gold and the other is silver. A box is chosen at random and from it a coin is chosen at random. If this coin is gold, what is the probability that it came from the box containing two gold coins?
16. Let A and B be events with probabilities $P[A]$ and $P[B]$.
 - a. Find $P[A \cup B]$ if A and B are independent.
 - b. Find $P[A \cup B]$ if A and B are mutually exclusive.
17. If A , B , and C are independent events such that $P(A) = 0.2$, $P(B) = 0.1$, and $P(C) = 0.4$, find $P(A \cup B \cup C)$.
18. Ten percent of items from a certain production line are defective. What is the probability that there is more than one defective item in a batch of n items?
19. Each of two persons tosses three fair coins. What is the probability that they obtain the same number of heads?
20. Consider a well-shuffled deck of cards consisting of 52 distinct cards, of which four are aces and four are kings.
 - a. Find the probability of obtaining an ace in the first draw.
 - b. Draw a card from the deck and look at it. What is the probability of obtaining an ace in the second draw? Does the answer change if you had not observed the first draw?
 - c. Suppose we draw 7 cards from the deck. What is the probability that the 7 cards include 3 aces? What is the probability that the 7 cards include 2 kings? What is the probability that the 7 cards include 3 aces and/or 2 kings?
 - d. Suppose that the entire deck of cards is distributed equally among four players. What is the probability that each player gets an ace?