# The astronomy of the age of geometric altars

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**Abstract**

Fire altars were an important part of ritual throughout the ancient world. Geometric ritual, often a part of the fire altars, was intimately connected with problems of mathematics and astronomy. Manuals of altar design from India explain the basis behind the reconciliation of the lunar and the solar years. This astronomy is based on the use of mean motions. Computation rules from *Vedāṅga Jyotiṣa*, an astronomy manual from the latter part of the second millennium B.C. that was used during the closing of the age of the altar ritual, are also described.

## 1 Introduction

It has long been argued that science had an origin in ritual. According to Plutarch (*Epicurum IX*) Pythagoras sacrificed an ox when he discovered the theorem named after him. This legend is, in all probability, false since Pythagoras was opposed to killing and sacrificing of animals, especially cattle (van der Waerden 1961, page 100). Nevertheless, this story frames the connection between ritual and science in the ancient world. Plutarch says elsewhere (*Quaestiones Convivii, VIII*, Quaest. 2.4) that the sacrifice of the bull was in connection with the problem of constructing a figure with

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the same area as another figure and a shape similar to a third figure. A. Seidenberg (1962, 1978) has sketched persuasive arguments suggesting that the birth of geometry and mathematics can be seen in the requirements of geometric ritual of precisely this kind. He shows how geometric ritual represented knowledge of the physical world through equivalences. But if geometric ritual represented a language that coded the knowledge of its times, it should have been used for astronomical knowledge as well. This question has recently been analyzed by the author (see Kak 1993a,b.c, 1994a,b) for the Indian context.

It is generally accepted that Hipparchus discovered precession in 127 B.C. The magnitude calculated by Hipparchus and accepted by Ptolemy was 1 degree in 100 years. The true value of this precession is about 1 degree in 72 years. Clearly the discovery of precession could not have been made based on observations made in one lifetime. The ancient world marked seasons with the heliacal rising of stars. So Hipparchus must have based his theory regarding precession on an old tradition. That the ancients were aware of the shift in the heliacal rising of stars with age was demonstrated by Giorgio de Santillana and Hertha von Dechend (1969) in their famous book *Hamlet's Mill* which appeared more than twenty five years ago. By uncovering the astronomical frames of myths from various ancient cultures, they showed that man's earliest remembrance of astral events goes back at least ten thousand years. Although, this does not mean that the principles behind the shifting of the astronomical frame were known at this early time, the analysis of the designs of Stonehenge, the pyramids, and other monuments establish that the ancients made careful astronomical observations. But no corroborative text, prior to the tablets from Babylon that date back to the middle of the first millennium B.C., is available.

Fire altars have been found in the third millennium cities of the Indus-Sarasvati civilization (Rao 1991) of India. The texts that describe their designs are conservatively dated to the first millennium B.C., but their contents appear to be much older. Basing his analysis on the Pythagorean triples in Greece, Babylon, and India, Seidenberg (1978) concludes that the knowledge contained in these texts—called *Śulba Sūtras*—goes back to at least 1700

B.C. More recent archaeological evidence, together with the astronomical references in the texts (Sastry 1985), suggests that this knowledge belongs to the third millennium B.C.

This article reviews the notions of equivalence by number and area that lay at the basis of the geometric altars. Issues related to the altar designs, and their astronomical significance, are summarized. We also consider Vedāṅga Jyotiṣa (VJ), an astronomical text that was in use during the times of the altar ritual. VJ has an internal date of c. 1350 B.C., give and take a couple of centuries, (Sastry 1985), obtained from its assertion that the winter solstice was at the asterism Śraviṣṭhā (Delphini). Recent archaeological discoveries (see summary in Kak 1994b) support such an early date, and so this book assumes great importance in the understanding of the earliest astronomy.

## 2 Altars and the Vedas

Fire altars were used extensively in several parts of Eurasia; for example, the Greeks had fire cults associated with Hephaistos and Hestia, whereas Rome had the cult of Vesta. However, records giving details of the geometric altar designs are available only from India. But as mentioned earlier, the drawing of figures of the same areas was an important part of altar design in India as well as Greece. Likewise, the altar ritual in Iran was very similar to the Indian one.

There was also a connection between monumental architecture and astronomy that can be seen from the temples and pyramids from Egypt, the temples of Mesopotamia, and megalithic monuments such as Stonehenge. Manuals of temple design from India spell this out most clearly. An Indic temple was a representation of the universe; a striking example of this is the temple at Angkor Wat.

Georges Dumézil (1988) has drawn attention to several striking parallels between the roles of the *brahmin*, the Indian fire-priest, and *flamen*, his Roman counterpart. The references by Plutarch regarding the significance of drawing figures related in area, and the similarity between the offices of the fire-priest in India and Greece and Rome, suggest that the ritual may have been similar.

This brings us to the Vedic times of India. Veda means knowledge in Sanskrit. The early Vedic times were characterized by the composition of hymns that were collected together in four books. The oldest of these books is the Rigveda; the one that deals with the performance of ritual is called Yajurveda.

## What are the Vedas?

The central idea behind the Vedic system is the notion of *connections* between the astronomical, the terrestrial, and the physiological. These connections were described in terms of number or other characteristics. An example is the 360 bones of the infant (which later fuse into the 206 bones of the adult) and the 360 days of the year, and so on. Although the Vedic books speak often about astronomical phenomena, it is only recently that the astronomical substratum of the Vedas has been examined (Kak 1994b).

My own researches have outlined the astronomy of the Indian fire altars of the Vedic times and shown that this knowledge was also coded in the organization of the Rigveda, which was taken to be a symbolic altar of hymns (Kak 1994b). The examination of the Rigveda is of unique significance since this ancient book has been preserved with incredible fidelity. This fidelity was achieved by remembering the text not only as a sequence of syllables (and words) but also through several different permutations of these syllables.

A.A. Macdonell, a major 19th century scholar of the Vedas, came to the following conclusion after studying the Rigvedic text and its indexing tradition:

> [It is] one of the most remarkable facts in the history of literature that a people ... have preserved its sacred book without adding or subtracting a single word for 2300 years, and that too chiefly by means of oral tradition. (Macdonell 1886, page xviii)

That the number of syllables and the verses of the Rigveda are according to an astronomical plan is claimed in other books of nearly the same antiquity such as the *Śatapatha Brāhmaṇa* (Kak 1992, 1994b). Rigveda may be considered an ancient *word monument*. It appears that the tradition, insisting that not a single syllable of the Rigveda be altered, arose from an attempt to be true to observed astronomical facts. Meanwhile, recent archaeological discoveries have also pushed back the dating of the Rigveda. Its new estimates of antiquity follow from the recent discoveries that date the drying up of the river Sarasvati, the pre-eminent river of the Rigvedic era, to around 1900 B.C. In other words, the astronomical characteristics of the Rigveda are to be dated to at least as early as 1900 B.C. The Vedāṅga Jyotiṣa has been, on linguistic grounds, dated to about five or six hundred years after the Rigveda; its internal date is thus in accord with the new chronology of the Rigveda.

## 3 Ritual and equivalence

Vedic ritual was generally performed at an altar. The altar design was based on astronomical numbers related to the reconciliation of the lunar and solar years. Vedic rites were meant to mark the passage of time. A considerable part of the ritual deals with altar construction. The fire altars symbolized the universe and there were three types of altars representing the earth, the space and the sky. The altar for the earth was drawn as circular whereas the sky (or heaven) altar was drawn as square. The geometric problems of circulature of a square and that of squaring a circle are a result of equating the earth and the sky altars. As we know these problems are among the earliest considered in ancient geometry.

### Equivalence by number

The altar ground where special ritual was conducted was called the *mahavedi*. This was an isosceles trapezoid having bases 24 and 30 and width 36. The sum of these numbers is 90, which was chosen since it represents one-fourth of the year. If the sum represents an example of equivalence by number, it is not clear why the shape of a trapezoid, with its specific dimensions, was chosen. But this shape generates many Pythagorean triples. On the mahavedi six small altars, representing space, and a new High Altar, *uttaravedi*, representing the sky were constructed.

The fire altars were surrounded by 360 enclosing stones, of these 21 were around the earth altar, 78 around the space altar and 261 around the sky altar. In other words, the earth, the space, and the sky are symbolically assigned the numbers 21, 78, and 261. Considering the earth/cosmos dichotomy, the two numbers are 21 and 339 since cosmos includes the space and the sky.

The main altar was built in five layers. The basic square shape was modified to several forms, such as falcon and turtle (Figure 1). These altars were built in five layers, of a thousand bricks of specified shapes. The construction of these altars required the solution to several geometric and algebraic problems (Sen and Bag 1983).

## Equivalence by area

The main altar was an area of $7\frac{1}{2}$ units. This area was taken to be equivalent to the nominal year of 360 days. Now, each subsequent year, the shape was to be reproduced with the area increased by one unit.

The ancient Indians spoke of two kinds of day counts: the solar day, and *tithi*, whose mean value is the lunar year divided into 360 parts. They also considered three different years: (1) *nakṣatra,* or a year of 324 days (sometimes 324 tithis) obtained by considering 12 months of 27 days each, where this 27 is the ideal number of days in a lunar month; (2) lunar, which is a fraction more than 354 days (360 tithis); and (3) solar, which is in excess of 365 days (between 371 and 372 tithis). A well-known altar ritual says that altars should be constructed in a sequence of 95, with progressively increasing areas. The increase in the area, by one unit yearly, in building progressively larger fire altars is 48 tithis which is about equal to the intercalation required to make the nakṣatra year in tithis equal to the solar year in tithis. But there is a residual excess which in 95 years adds up to 89 tithis; it appears that after this period such a correction was made. The 95 year cycle corresponds to the tropical year being equal to 365.24675 days. The cycles needed to harmonize various motions led to the concept of increasing periods and world ages.

## 4 The Rigvedic altar

The number of syllables in the Rigveda confirms the textual references that the book was to represent a symbolic altar. According to various early texts, the number of syllables in the Rigveda is 432,000, which is the number of *muhurtas* (1 day = 30 muhurtas) in forty years. In reality the syllable count is somewhat less because certain syllables are supposed to be left unspoken.

The verse count of the Rigveda can be viewed as the number of *sky* days in forty years or $261 \times 40 = 10,440$, and the verse count of all the Vedas is $261 \times 78 = 20,358$. The detailed structure of the Rigveda also admits of other astronomical interpretations (Kak 1994b); these include the fact that the sun is about 108 sun-diameters and the moon is about 108 moon-diameters away from the earth; this can be easily established by checking that the angular size of a pole that is 108 lengths away from the observer equals that of the sun or the moon. There also exists compelling evidence that the periods of the planets had been obtained.

The number 108, the number of sun-steps away from the earth, assumed great symbolic significance. The 108 beads in the rosary are these 108 steps that represent the path from earth to heaven. In the temple of Angkor Wat the complex is surrounded by a moat which is bridged by roads leading from five gates. "Each of these roads is bordered by a row of huge stone figures, 108 per avenue" (Santillana and Dechend 1969). "When measured in Khmer *hat*, the 617-foot length of the bridge corresponds to the 432,000 years of an age of decadence, and the 2,469 feet between the first steps of the bridge and the threshold of the temple's center represent the 1,728,000 years of a golden age" (White 1982). So this temple complex merely represents an ancient model.

## 5 The motions of the sun and the moon

The *Vedāṅga Jyotiṣa* (VJ) (Sastry 1985) describes the mean motions of the sun and the moon. This manual is available in two recensions: the earlier Rigvedic VJ (RVJ) and the later Yajurvedic VJ (YVJ). RVJ has 36 verses and YVJ has 43 verses.

The measures of time used in VJ are as follows:

1 lunar year = 360 tithis
1 solar year = 366 solar days
1 day = 30 muhūrtas
1 muhūrta = 2 nāḍikās
1 nāḍikā = $10\frac{1}{20}$ kalās
1 day = 124 aṁśas (parts)

1 day = 603 kalās

Furthermore, five years were taken to equal a *yuga*. A ordinary yuga consisted of 1,830 days. An intercalary month was added at half the yuga and another at the end of the yuga.

What are the reasons for the use of a time division of the day into 603 kalās? This is explained by the assertion VJ 29 that the moon travels through 1,809 nakṣatras in a yuga. Thus the moon travels through one nakṣatra in $1\frac{7}{603}$ sidereal days because

$$1,809 \times 1\frac{7}{603} = 1,830.$$

Or the moon travels through one nakṣatra in 610 kalās. Also note that 603 has 67, the number of sidereal months in a yuga, as a factor The further division of a kalā into 124 kāṣṭhās was in symmetry with the division of a yuga into 62 synodic months or 124 fortnights (of 15 tithis), or parvans. A parvan is the angular distance travelled by the sun from a full moon to a new moon or vice versa.

The ecliptic was divided into twenty seven equal parts, each represented by a nakṣatra or constellation. The VJ system is a coordinate system for the sun and the moon in terms of the 27 nakṣatras. Several rules are given so that a specific tithi and nakṣatra can be readily computed.

> The number of risings of the asterism Śraviṣṭhā in the yuga is the number of days plus five (1830+5 = 1835). The number of risings of the moon is the days minus 62 (1830-62 = 1768). The total of each of the moon's 27 asterisms coming around 67 times in the yuga equals the number of days minus 21 (1830-21 = 1809). (YVJ 29)

> The moon is conjoined with each asterism 67 times during a yuga. The sun stays in each asterism $13\frac{5}{9}$ days. (RVJ 18, YVJ 39)

The explanations are straightforward. The sidereal risings equals the 1,830 days together with the five solar cycles. The lunar cycles equal the 62 synodic months plus the five solar cycles. The moon's risings equal the risings of Śraviṣṭhā minus the moon's cycles.

This indicates that the moon was taken to rise at a mean rate of

$$\frac{1,830}{1,768} = 24 \textit{ hours and } 50.4864 \textit{ minutes.}$$

# 6 Computation of tithis, nakṣatras, kalās

Although we have spoken of a mean tithi related through the lunar year equalling 360 tithis, the determination of a tithi each day is by a calculation of a shift of 12° with respect to the sun. In other words, in 30 tithis it will cover the full circle of 360°. But the shift of 12° is in an irregular manner and the duration of the tithi can vary from day to day. As a practical method a mean tithi can be defined by a formula. In terms of kalās a tithi is approximately 593 kalās. VJ takes it to be 122 parts of the day divided into 124 parts (RVJ 22, YVJ 37, 40).

Since the calendar was calibrated by nakṣatras, tithis were figured by a rule and not in a precise mean manner.

Each yuga was taken to begin with the asterism Śraviṣṭhā and the synodic month of Māgha, the solar month Tapas and the bright fortnight (parvan), and the northward course of the sun and the moon (RVJ 5-6; YVJ 6-7). The intercalary months were used in a yuga. But since the civil year was 366 days, or 372 tithis, it was necessary to do further corrections. As shown in the earlier section, a further correction was performed at 95 year, perhaps at multiples of 19 years.

The day of the lunar month corresponds to the tithi at sunrise. A tithi can be lost whenever it begins and ends between one sunrise and the next. Thus using such a mean system, the days of the month can vary in length.

## Rule on end of parvan

The determination of the exact ending of the synodic fortnight (parvan) is important from the point of view of the performance of ritual. Let $p$ be the parvans that have elapsed from the beginning of the yuga. Since each parvan has 1,830 parts, the number of parts, $b$, remaining in the day at the end of $p$ parvans is:

$$b = 1830\ p\ mod\ 124.$$

Now consider

$$p\ mod\ 4 = \alpha,$$

and

$$1830\ mod\ 31 = 1.$$

By multiplying the two modular equations, it can be easily shown that

$$b = (1829 \times \alpha + p) mod\ 124.$$

By substituting the values $\alpha = 1, 2, 3$ we get the YVJ 12 rule:

When $\alpha = 1$, b = p + 93 mod 124;
when $\alpha = 2$, b = p + 62 mod 124;
when $\alpha = 3$, b = p + 31 mod 124.

## Rule on nakṣatra parts

The nakṣatra part of the sun at the end of the $p$th parvan, $s$, is clearly:

$$s = 135\ p\ mod\ 124.$$

This is because 124 parvans equal the 135 nakṣatra segments for the sun at the end of the yuga of 5 years. Let $p = 12 \times q + r$. Then we can write:

$$135\ p\ mod\ 124 = 11 \times (12q + r) = 8q + 11r\ mod\ 124.$$

This is the rule described to compute the nakṣatras of the sun (RVJ 10, YVJ 15).

If the moon is full, it will be in opposition to the sun and, therefore, $13\frac{1}{2}$ segments, or 13 nakṣatras and 62 parts away. So the rule further states that for a full moon its nakṣatra parts are computed by adding 62 to the parts obtained for the sun. This can be seen directly by noting that the nakṣatra parts of the moon, $m$, will be according to:

$$m = 1809\ p\ mod\ 124.$$

This leads to the equation:

$$m = 8q + 73r\ mod\ 124.$$

This is in excess from $s$ by $62r\ mod\ 124$, which is 62 when $p$ is odd.

## Moon nakṣatra in kalās

Since 124 parvans correspond to 1,809 or $67 \times 27$ nakṣatras, 17 parvans correspond to $248 + \frac{1}{124}$ nakṣatras. Now the moon passes through each nakṣatra in 610 kalās, therefore the 248 days correspond to $\frac{248 \times 610}{603}$ days; this equals 250 days and 530 kalās. If we assume that we are just one part short of the 16th parvan, we have its modular relationship with 530 kalās. For 8 nakṣatra parts short, this corresponds to $530 \times 8\ mod\ 603 = 19$ kalās. Each part is -73 kalās. This rule is in RVJ 11 and YVJ 19.

## Other rules and accuracy

There are other rules of a similar nature which are based on the use of congruences. These include rules on hour angle of nakṣatras, time of the day at the end of a tithi, time at the beginning of a nakṣatra, correction for the sidereal day, and so on. But it is clear that the use of mean motions can lead to discrepancies that need to be corrected at the end of the yuga.

The framework of VJ has approximations built into it such as consideration of the civil year to be 366 days and the consideration of a tithi as being equal to $\frac{122}{124}$ of a day. The error between the modern value of tithi and its VJ value is:

$$\frac{354.367}{360} - \frac{122}{124}$$

which is as small as $5 \times 10^{-4}$. This leads to an error of less than a day in a yuga of five years.

The constructions of the geometric altars as well as the Vedic books that come centuries before VJ (Kak 1994b) confirm that the Vedic Indians knew that the year was more than 365 days and less than 366 days. The VJ system could thus have only served as a framework. It appears that there were other rules of missing days that brought the calendar into consonance with the reality of the nakṣatras at the end of the five year yuga and at the end of the 95 year cycle of altar construction.

Mean motion astronomy can lead to significant discrepancy between true and computed values. The system of intercalary months introduced further irregularity into the system. This means that the conjunction between the sun and the moon that was assumed at the beginning of each yuga became more and more out of joint until such time that the major extra-yuga corrections were made. This is perhaps the reason why the Indian books do not describe the location of the junction stars of the nakṣatras very accurately.

# 7 From altar astronomy to classical astronomy

Classical Indian astronomy arose after the close interaction between the Indians and the Greeks subsequent to the invasion of the borders of India by Alexander the Great (323 B.C.). The existence of an independent tradition of observation of planets and a theory thereof as shown by geometric altars, the Rigvedic code, and the VJ helps explain the puzzle why classical Indian astronomy uses many constants that are different from that of the Greeks. This confirms the thesis that although classical Indian astronomy developed in knowledge of certain Greek works, the reason why it retained its characteristic form was because it was based on an independent, old tradition.

The astronomy of the third and the second millennium B.C. can provide the context in which the developments of Babylonian, Chinese, Greek, and the later Indian astronomy can be examined. It appears that certain features of the earliest Babylonian astronomy can be derived from an altar astronomy that may have been widely known in the ancient world, but whose records are now available only in the Indian texts.

It also raises the question of an analysis of the altars and monuments from Babylon, Greece, and Rome to examine their designs. Likewise, the references in the Greek literature to geometric problems related to areas need to be investigated further for their astronomical significance.

Later religious architecture, both in the east and the west, became more abstract but its astronomical inspiration was never hidden. In Europe cathedrals were a representation of the vault of heavens. In India the temple architecture, as spelt out in the manuals of the first centuries A.D. (see, for example, Kramrisch 1946), symbolizes the sky where in addition to equivalence by number or area, equivalence by category was considered. The temple platform was divided into 64 or 81 squares (Figure 2). In the case of the 64-squared platform, the outer 28 squares represented the 28 lunar mansions of the Indic astronomy. For the 81-squared platform, the outer 32 squares were taken to represent the lunar mansions and the four planets who rule over the equinoxial and solstitial points. Stella Kramrisch, the renowned scholar of the Indian temple architecture, has also argued that another measure in the temple was that of 25,920, the number of years in the period of the precession of the equinoxes. Whether the precessional figure was received by the Indians from the Greeks or obtained independently is not clear.

## 8 On observation in the ancient world

As mentioned in Section 2, the ancients were aware of the parallels in the astronomical, terrestrial and physiological phenomena. The Vedic books are based on the ideology of such connections.

One can see a plausible basis behind the equivalences. Research has shown that all life comes with its inner clocks. Living organisms have rhythms that are matched to the periods of the sun or the moon. For example, the potato has a variation in its metabolic processes that is matched to the sidereal day, the 23-hour 56-minute period of rotation of the earth relative to the fixed stars. The cicadas come in many species including ones that appear yearly in midsummer. The best-known amongst the others are those that have 13-year and 17-year periods. There are quite precise biological clocks of 24-hour (according to the day), 24 hour 50 minutes (according to the lunar day since the moon rises roughly 50 minutes later every day) or its half representing the tides, 29.5 days (the period from one new moon to the next), and the year. Monthly rhythms, averaging 29.5 days, are reflected in the reproductive cycles of many marine plants and those of animals. It has been claimed that there are others that correspond to the periods of the planets. There are other biological periodicities of longer durations.

The use of the mean motions requires continual corrections. The constructions of the geometric altars indicates that such corrections were made on a regular basis. The corrected mean motion astronomy of the geometric altars and the VJ appears to have served a useful calendric purpose.

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