Automatic Performance Tuning and Analysis of Sparse Triangular Solve

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Berkeley Benchmarking and OPtimization (BeBOP) Project

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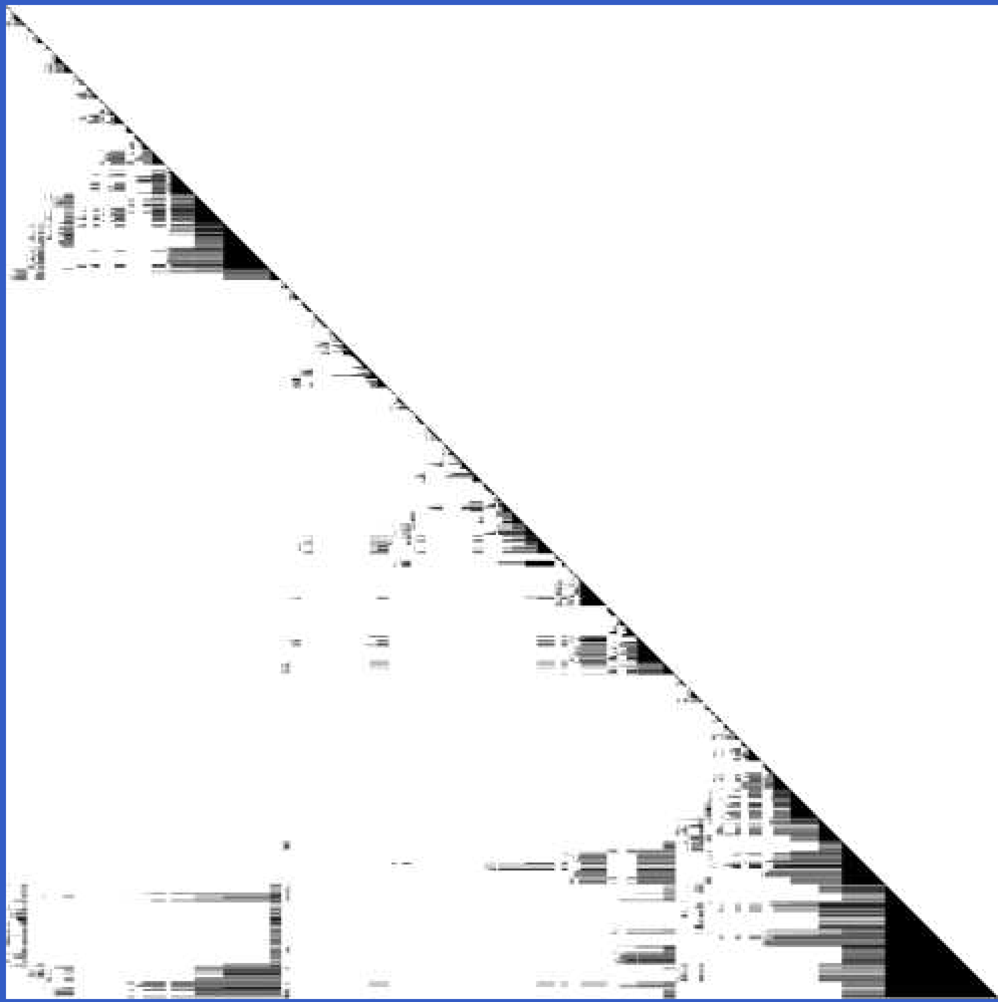
Context: High-Performance Libraries

- Application performance dominated by a few *computational kernels*
  - Solving PDEs (linear algebra ops)
  - Google (sparse matrix-vector multiply)
  - Multimedia (signal processing)
- Performance tuning today
  - Vendor-tuned standardized libraries (*e.g.*, BLAS)
  - User tunes by hand
- Automated tuning for dense linear algebra, FFTs, . . .
  - PHiPAC/ATLAS (dense linear algebra)
  - FFTW/SPIRAL/UHFFT (signal processing)

Problem Area: Sparse Matrix Kernels

- Performance issues in sparse linear algebra
  - High bandwidth requirements and poor instruction mix
  - Depends on architecture, kernel, and *matrix*
  - How to select data structures, algorithms? at run-time?
- Approach to automatic tuning: for each kernel,
  - *Identify* and *generate* a space of implementations
  - *Search* (models, experiments) to find the fastest one
- Early success: SPARSITY (Im & Yelick '99) for sparse matrix-vector multiply (SpM$\times$V)
- This talk: *Sparse triangular solve* (SpTS), arising in sparse Cholesky and LU factorization (uniprocessor)

Sparse Triangular Matrix Example



- `raefsky4` (structural problem) + SuperLU 2.0 + colmmd
- Dimension: 19779
- No. non-zeros: 12.6 M
- Dense trailing triangle:
  - dim=2268
  - 20% of total nnz

Idea: Sparse/Dense Partitioning

Partition the matrix into sparse (L_1, L_2) and dense (L_D) parts:

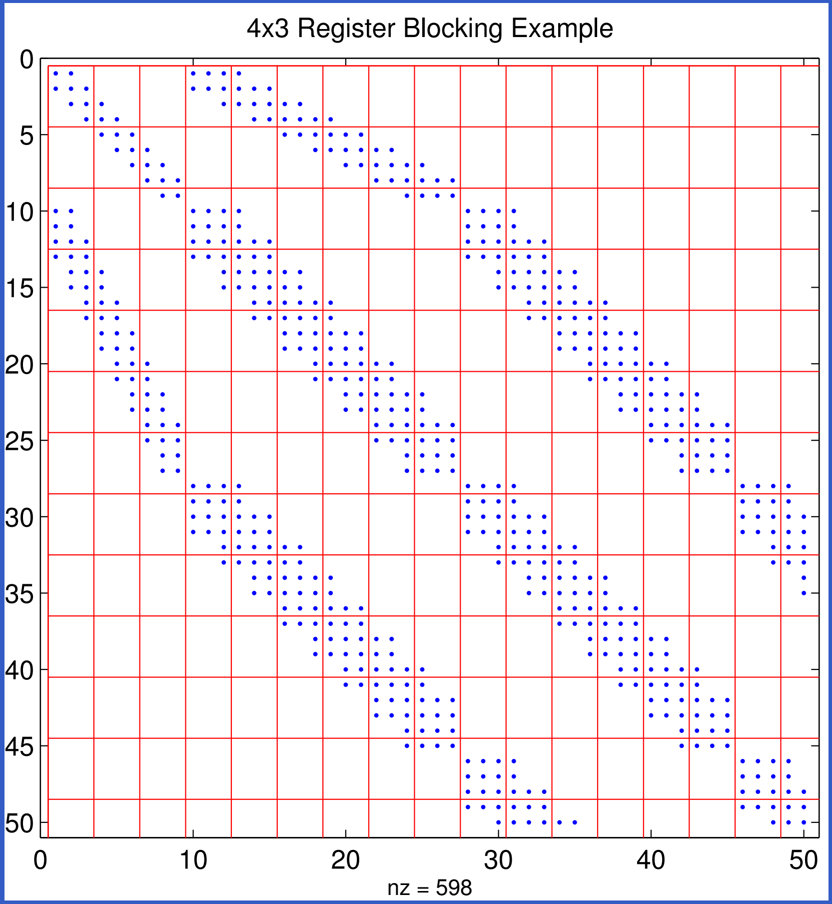
$$\begin{pmatrix} L_1 & \\ L_2 & L_D \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Leads to 1 SpTS, 1 SpM$\times$V, and 1 Dense TS:

$$\begin{aligned} L_1 x_1 &= y_1 && (1) \\ \hat{y}_2 &= y_2 - L_2 x_1 && (2) \\ L_D x_2 &= \hat{y}_2 && (3) \end{aligned}$$

SPARSITY optimizations for (1)–(2); tuned BLAS for (3).

Register Blocking (SPARSITY)



- Store $r \times c$ dense blocks
- Multiply/solve block-by-block
- Fill in explicit zeros
- 1.3x–2.5x speedup on FEM matrices (SpM$\times$V)
- Reduced storage overhead over, *e.g.*, CSR
- Block ops are fully unrolled – improves register reuse
- Trade-off extra computation for efficiency

Tuning Parameter Selection

- Parameters: *switch point*, s, and *register block size*, b
- Off-line profiling
  - Benchmark routines on synthetic data
  - Only needed *once* per architecture
- At run-time (when matrix is known)
  - Determine or estimate matrix properties (*e.g.*, fill ratio, size of trailing triangle)
  - Combine with data collected off-line
  - Convert to new data structure
- In practice, total run-time cost to select and reorg: *e.g.*, 10–30 naïve solves on Itanium

Performance Bounds

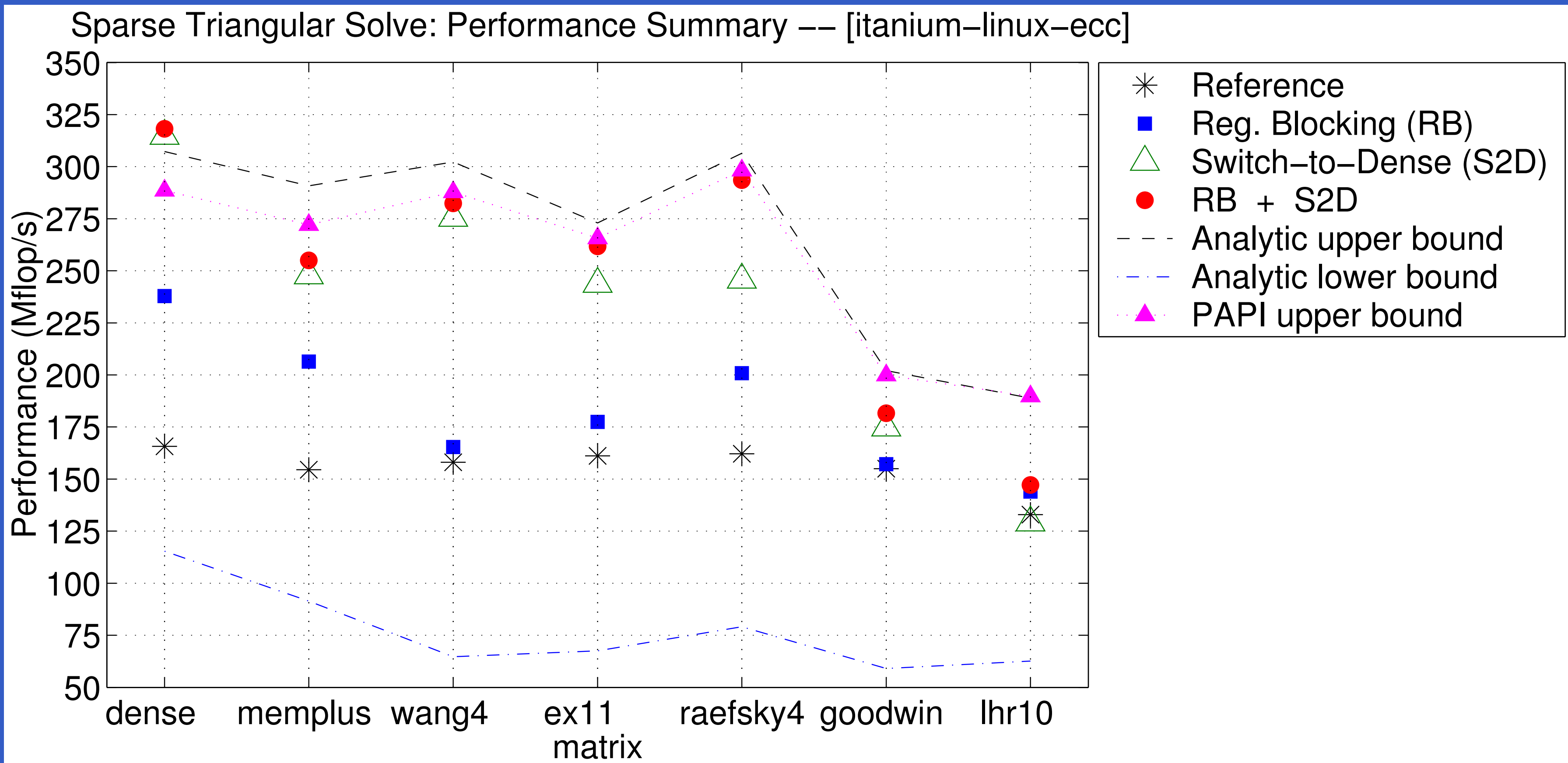
- *Upper-bounds* on performance (Mflop/s)?
- Flops: 2 * (number of non-zeros) - (dimension)
- Full latency cost model of execution time:

$$T(s,b) = \sum_{i=1}^{\kappa-1} H_i(s,b)\alpha_i + M_\kappa(s,b)\alpha_{\text{mem}} \tag{4}$$

- Lower bound on misses: ignore conflict misses on vectors

$$M_{\text{lower}}^{(i)}(s,b) = \frac{1}{l_i}\left[k f_b\left(1+\frac{1}{\gamma b^2}\right) + \frac{1}{\gamma}\left(\left\lceil\frac{n}{b}\right\rceil + 1\right) + \left(2n + \frac{(n-s)\left((n-s)+1\right)}{2}\right)\right] \tag{5}$$

Performance Results: Intel Itanium



Conclusions and Directions

- Limits of “low-level” tuning are near
  - Can we approach bandwidth limits?
  - Other kernels? $A^T Ax$, $A^k x$, RAR^T
  - Other structures? multiple vectors, symmetry, reordering
- Interface from/to libraries and applications?
  - Leverage existing generators (*e.g.*, Bernoulli)
  - Hybrid on-line, off-line optimizations
- SpTS-specific future work
  - symbolic structure; other fill-reducing orderings
  - refinements to switch point selection
  - Incomplete Cholesky and LU preconditioners

Related Work (1/R)

- Automatic tuning systems
  - PHiPAC [BACD97], ATLAS [WPD01], SPARSITY [Im00]
  - FFTW [FJ98], SPIRAL [PSVM01], UHFFT [MMJ00]
  - MPI collective ops (Vadhiyar, *et al.* [VFD01])
- Code generation
  - FLAME [GGHvdG01]
  - Sparse compilers (Bik [BW99], Bernoulli [Sto97])
  - Generic programming (Blitz++ [Vel98], MTL [SL98], GMCL [Neu98], . . .)
- Sparse performance modeling
  - Temam and Jalby [TJ92]
  - White and Sadayappan [WS97]
  - Navarro [NGLPJ96], Heras [HPDR99], Fraguela [FDZ99]

Related Work (2/R)

- Compilers (analysis and models); run-time selection
  - CROPS (UCSD/Carter, Ferrante, *et al.*)
  - TUNE (Chatterjee, *et al.*)
  - Iterative compilation (O'Boyle, *et al.*, 1998)
  - Broadway (Guyer and Lin, '99)
  - Brewer ('95); ADAPT (Voss, 2000)
- Interfaces: Sparse BLAS; PSBLAS; PETSc
- Sparse triangular solve
  - SuperLU/MUMPS/SPOOLES/UMFPACK/PSPASES…
  - Approximation: Alvarado ('93); Raghavan ('98)
  - Scalability: Rothberg ('92;'95); Gupta ('95); Li, Coleman ('88)

—End—

Tuning Parameter Selection

- First, select *switch point*, s; at run-time:
  - Assume matrix in CSR format on input
  - Scan bottom row from diag. until two consecutive zeros found
  - Fill vs. efficiency trade-off
- Then, select *register block size*, b
  - Maximize, over all b,

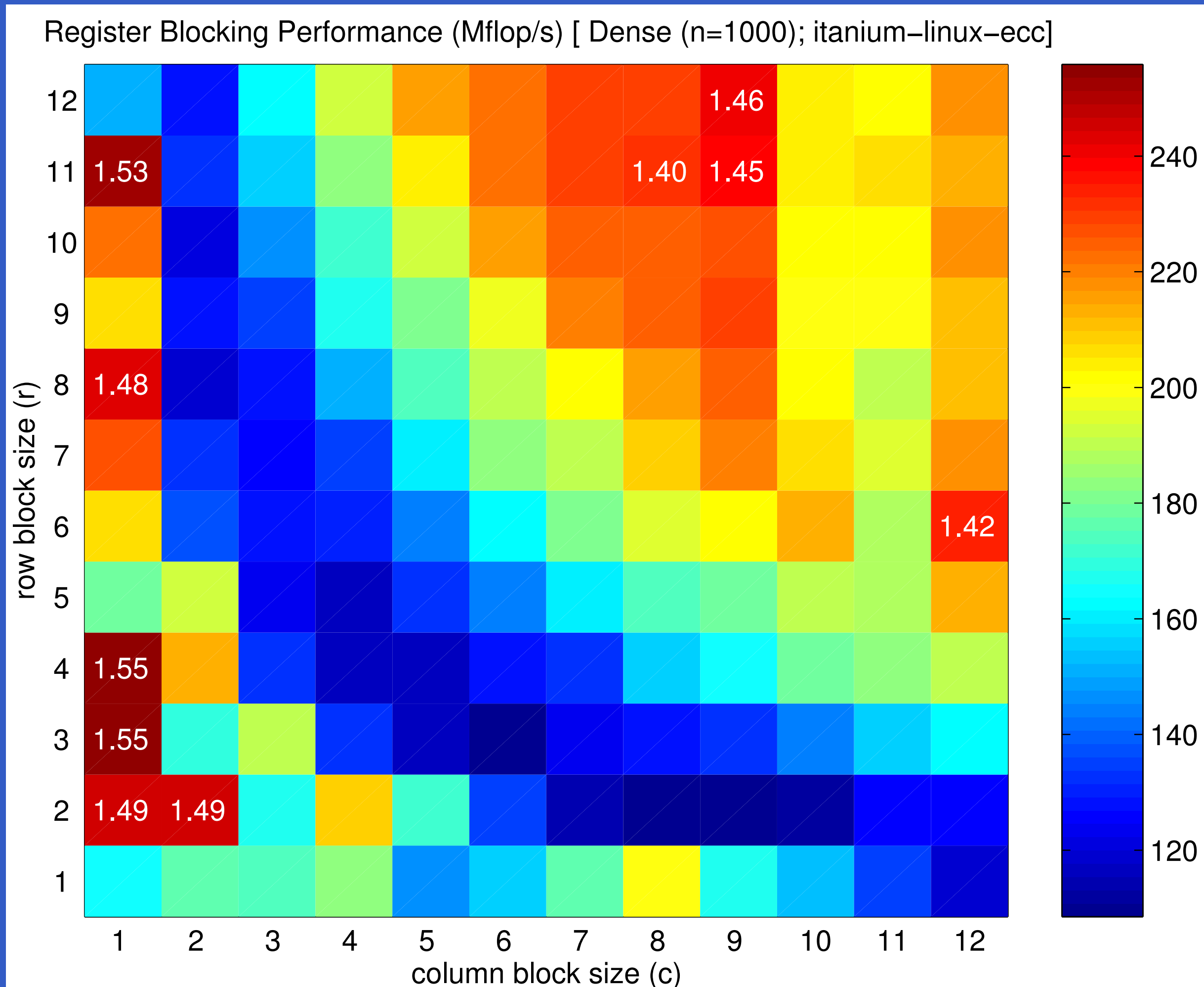
$$\frac{\text{Dense matrix in sparse format performance (Mflop/s) at } b}{\text{Estimated fill ratio at } b} \tag{6}$$

- Total cost to select and reorg.: *e.g.*, 10–30 naïve solves on Itanium

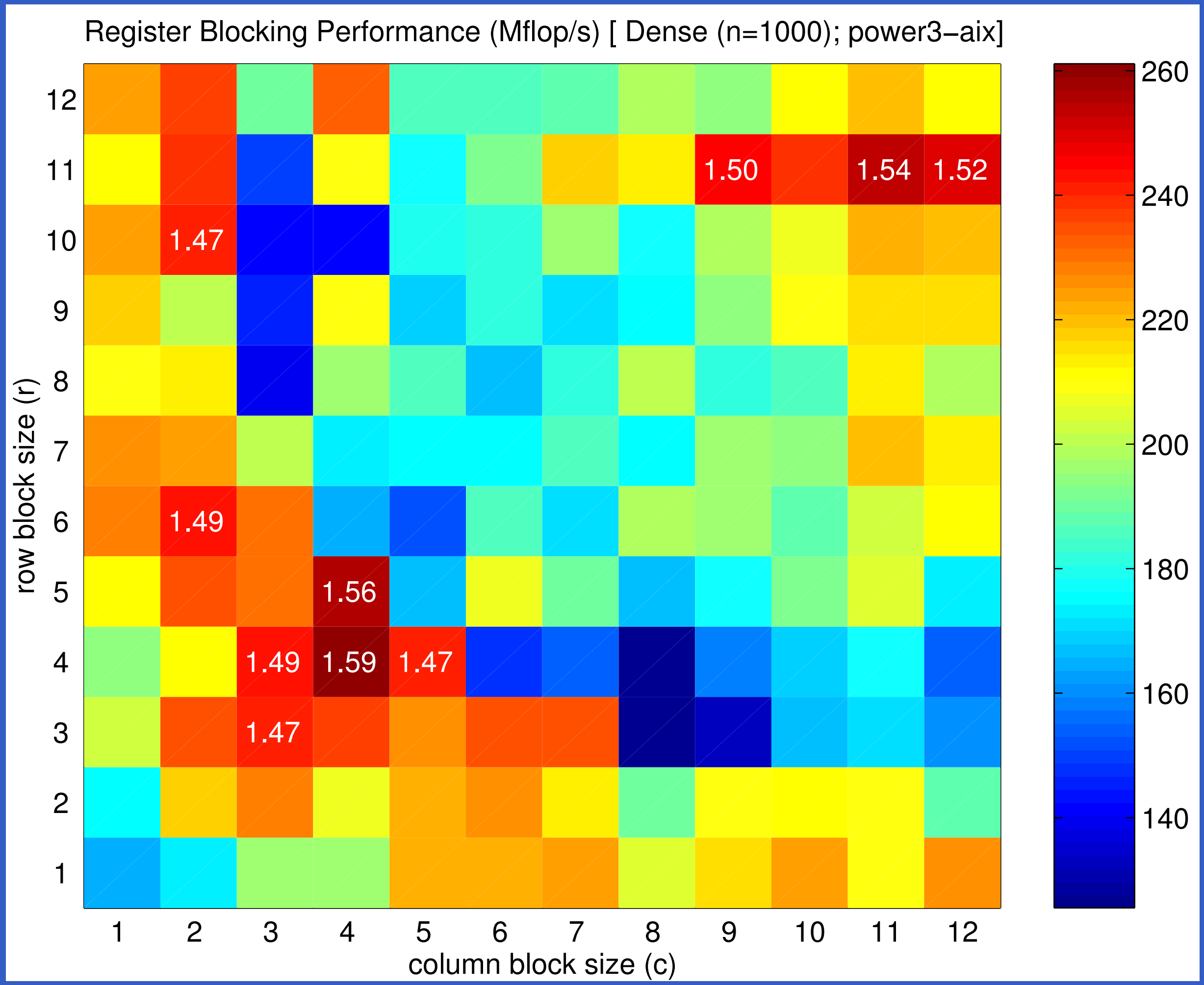
Matrix Benchmark Suite

Name	Application Area	Dim.	Nnz in L	Dense Trailing Triangle Dim.	Dense Trailing Triangle Density	Dense Trailing Triangle % Total Nnz
dense	Dense matrix	1000	500k	1000	100.0%	100.0%
memplus	Circuit simulation	17758	2.0M	1978	97.7%	96.8%
wang4	Device simulation	26068	15.1M	2810	95.0%	24.8%
ex11	Fluid flow	16614	9.8M	2207	88.0%	22.0%
raefsky4	Structural mechanics	19779	12.6M	2268	100.0%	20.4%
goodwin	Fluid mechanics	7320	1.0M	456	65.9%	6.97%
lhr10	Chemical processes	10672	369k	104	96.3%	1.43%

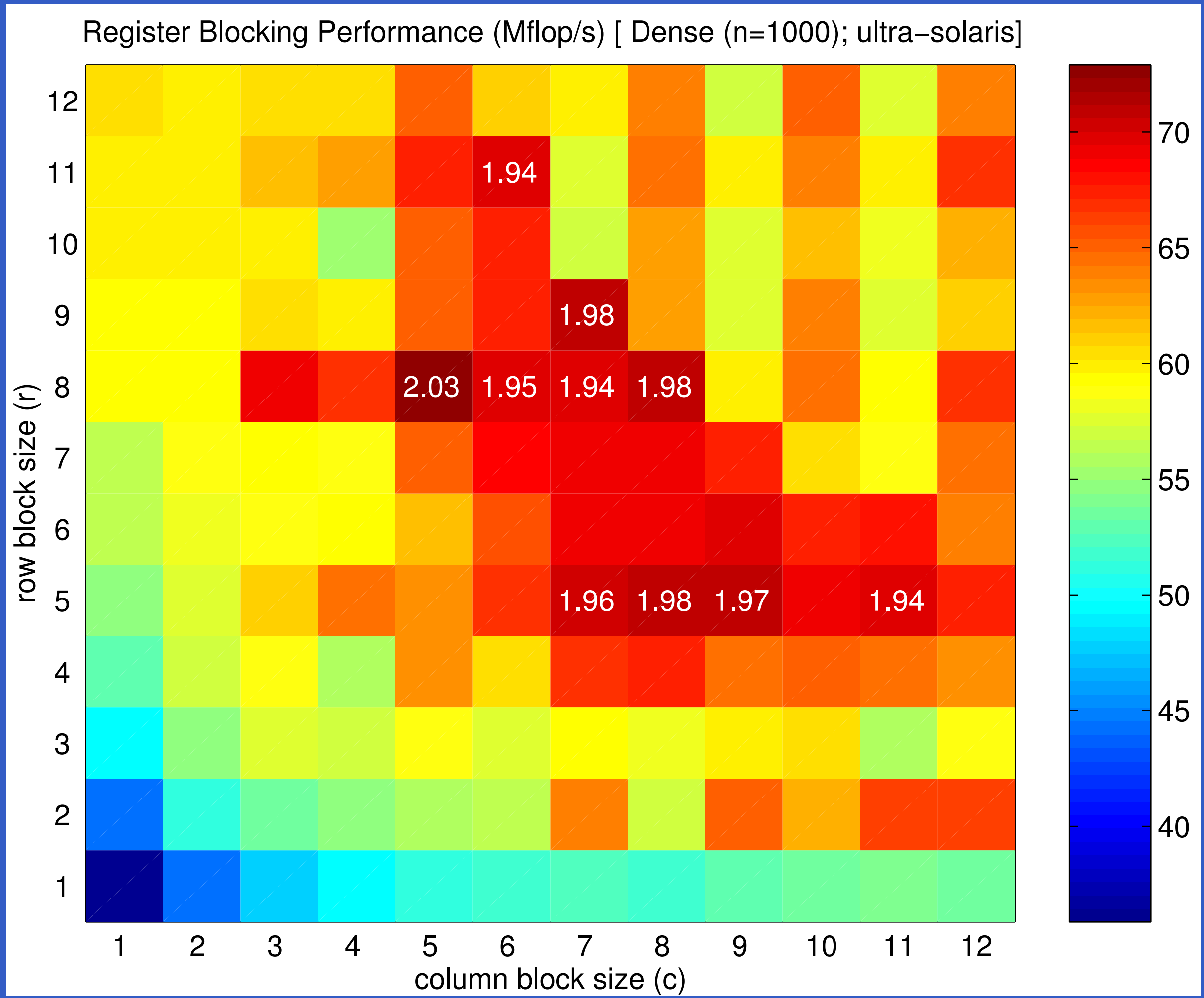
Register Profile (Intel Itanium)



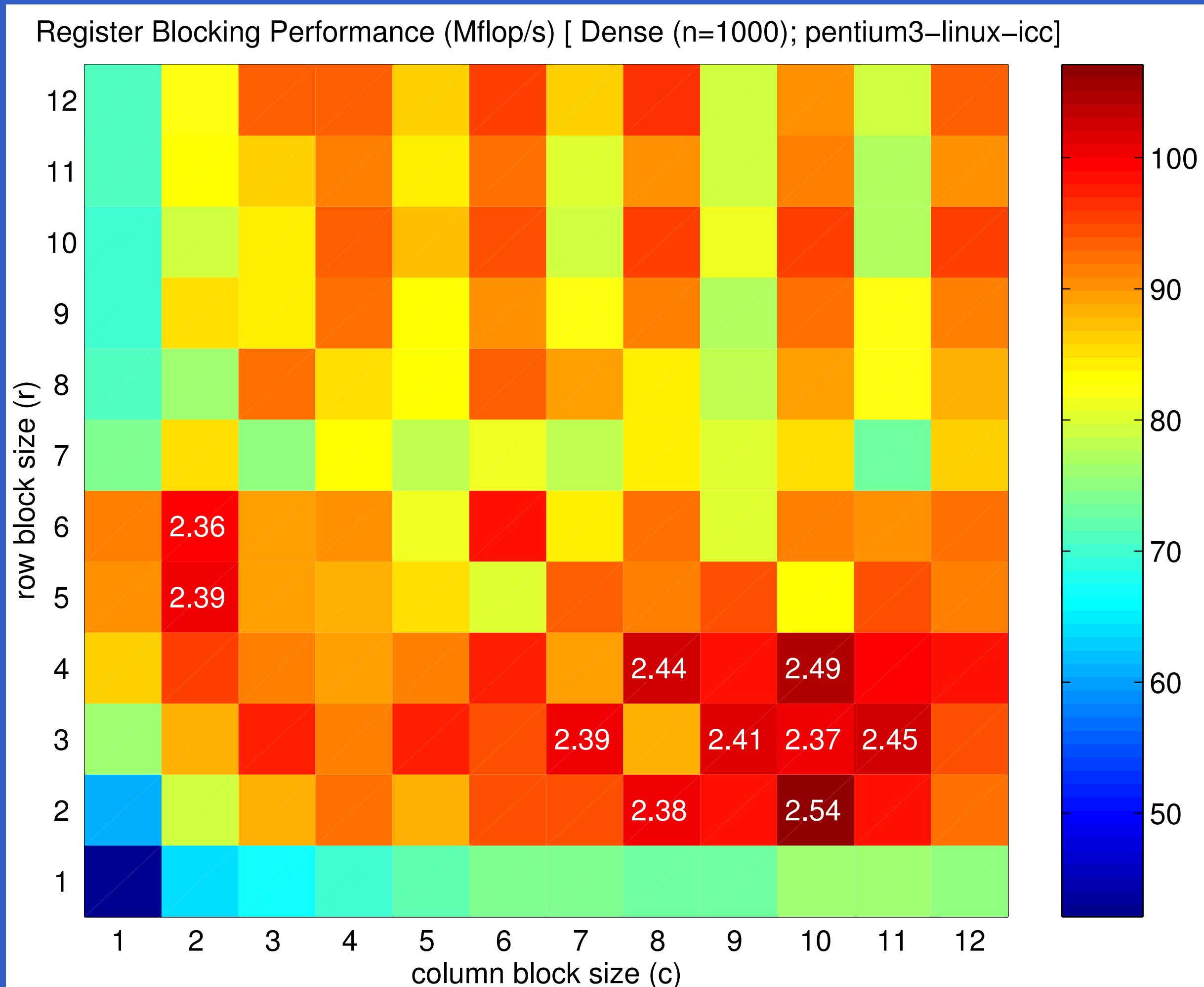
Register Profile (IBM Power3)



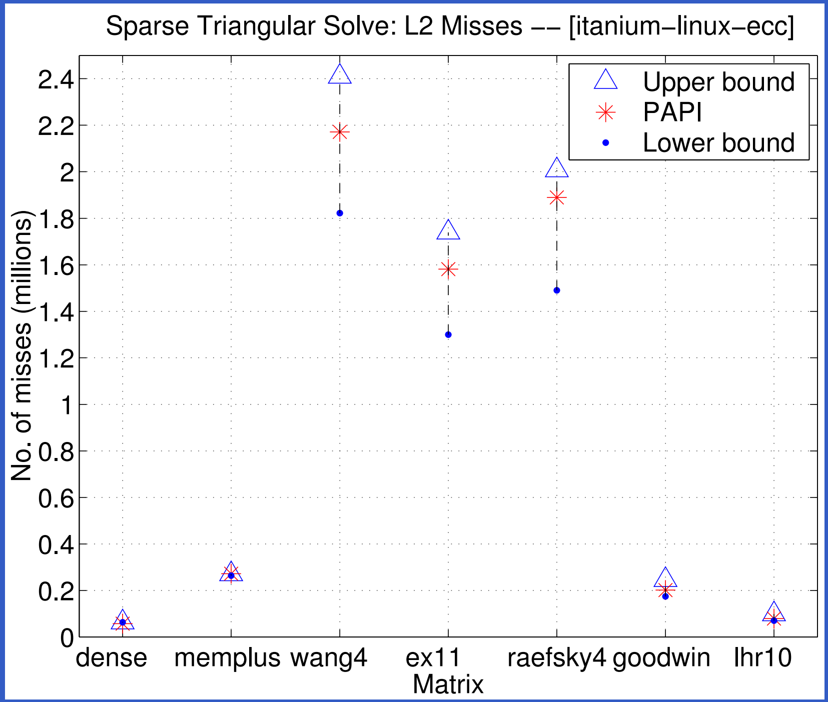
Register Profile (Sun Ultra 2i)

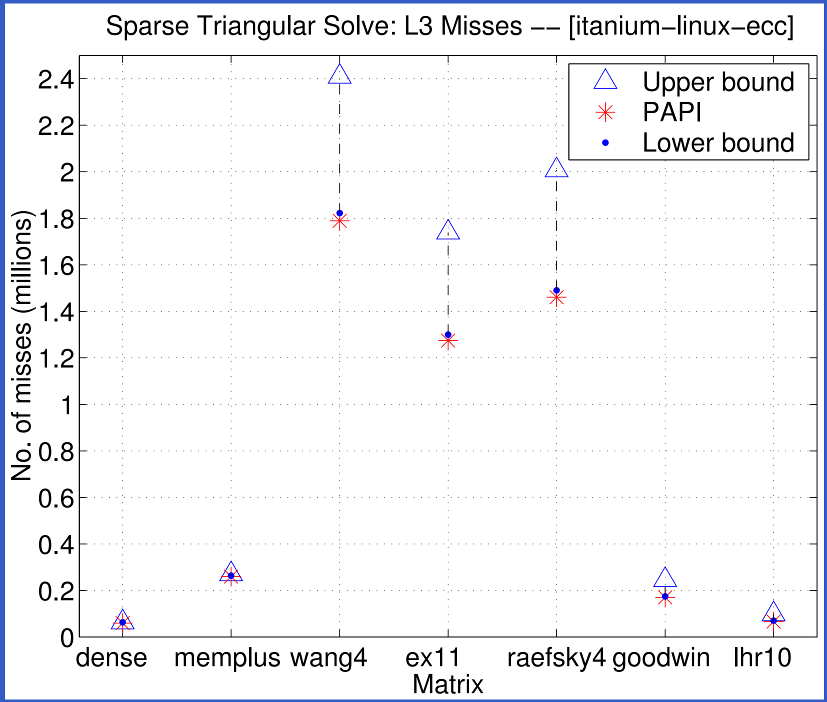


Register Profile (Intel Pentium III)

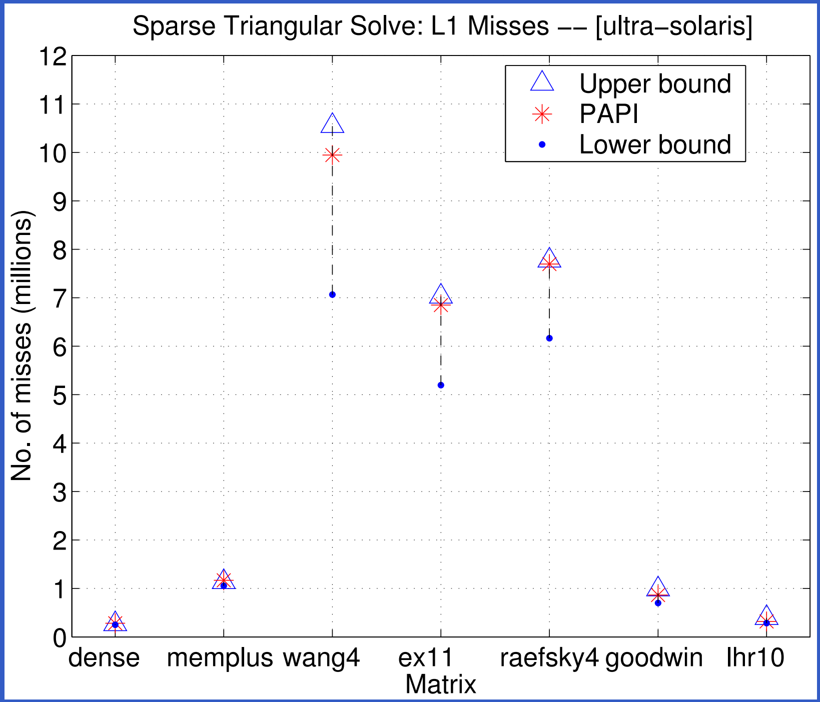


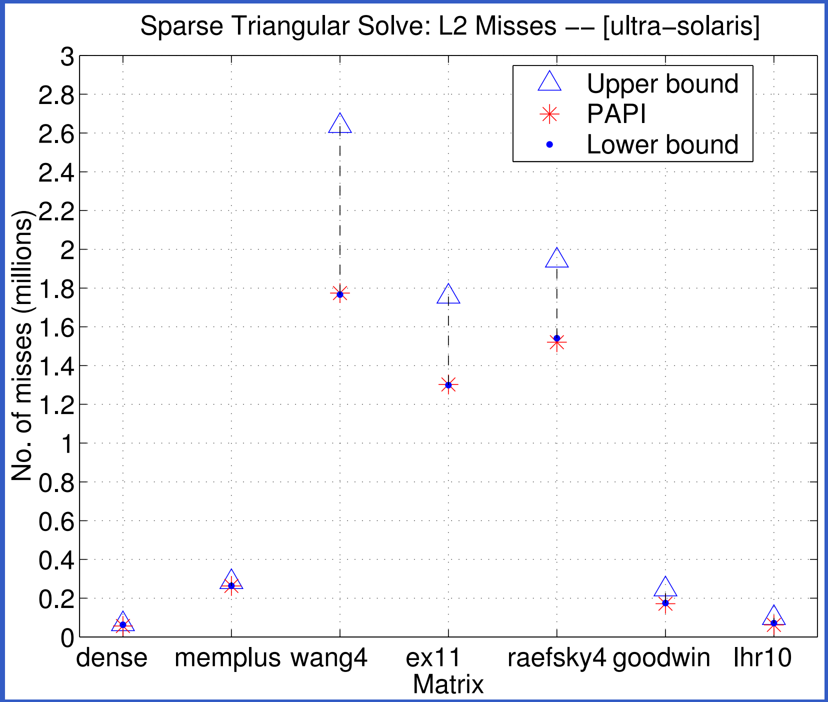
Miss Model Validation: Intel Itanium



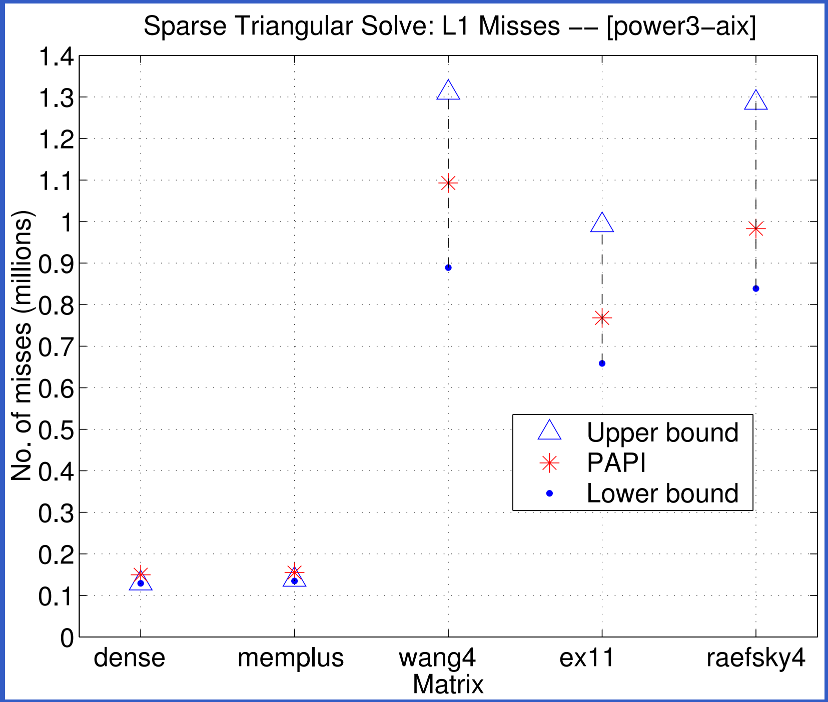


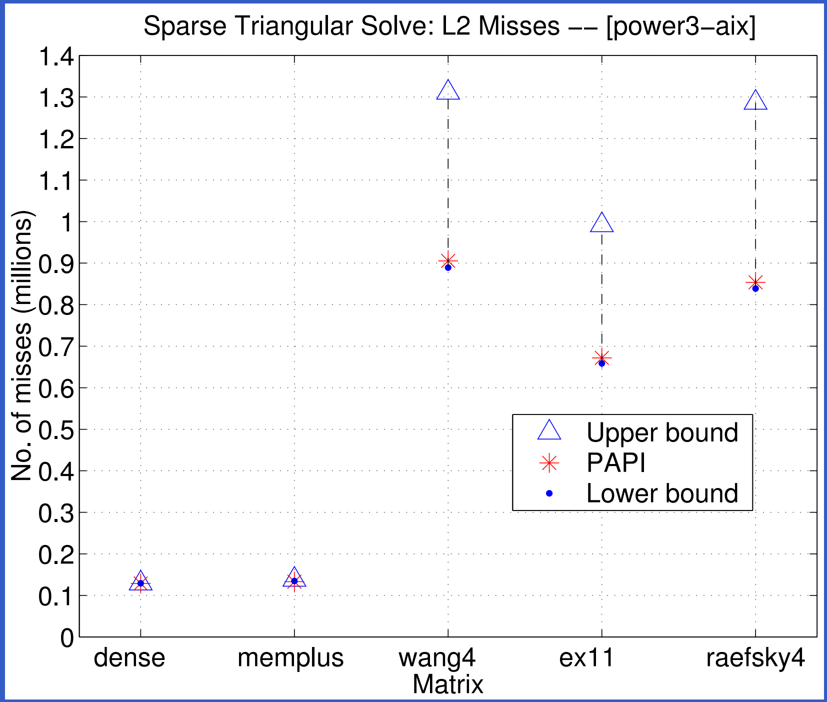
Miss Model Validation: Sun Ultra 2i



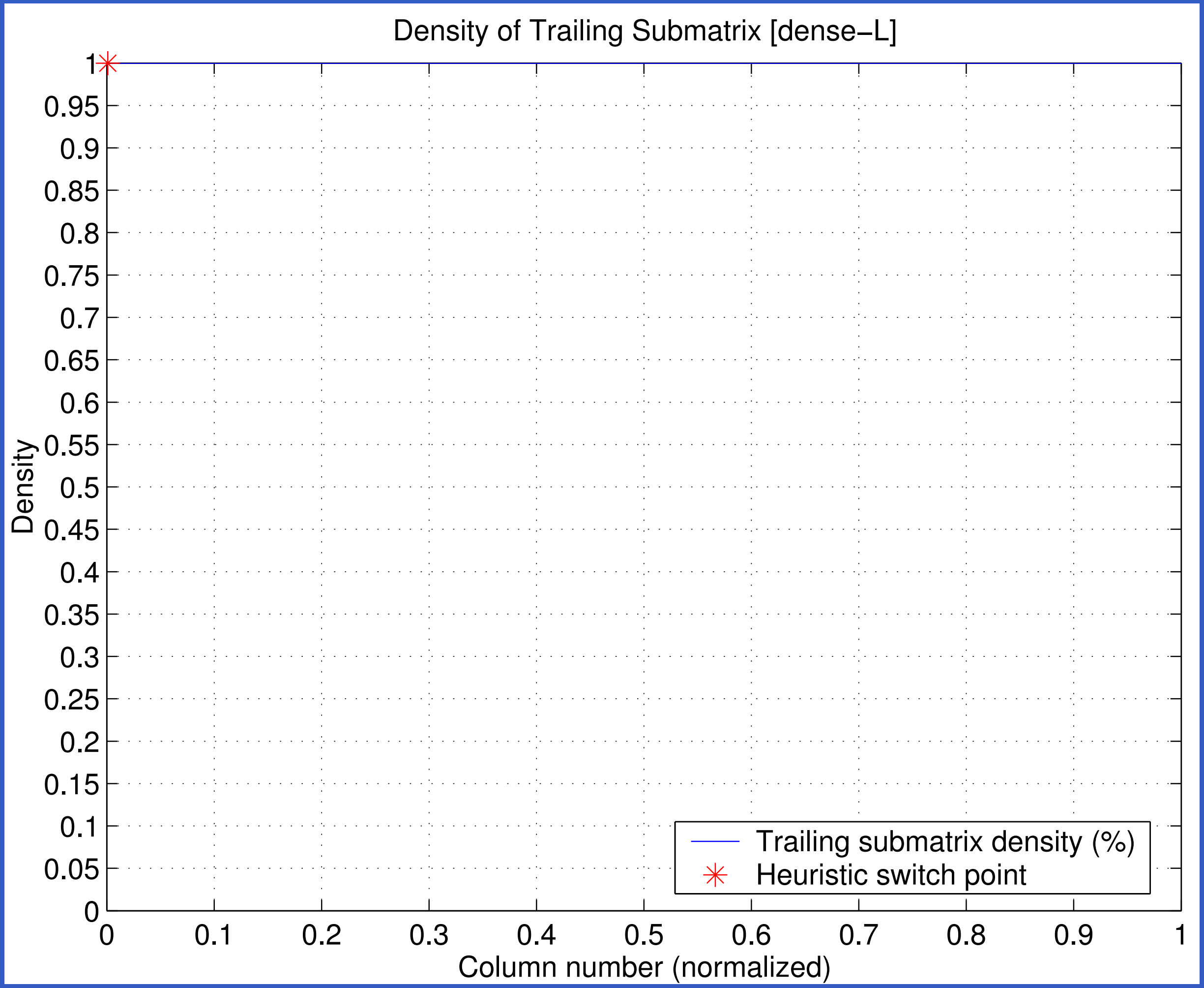


Miss Model Validation: IBM Power3

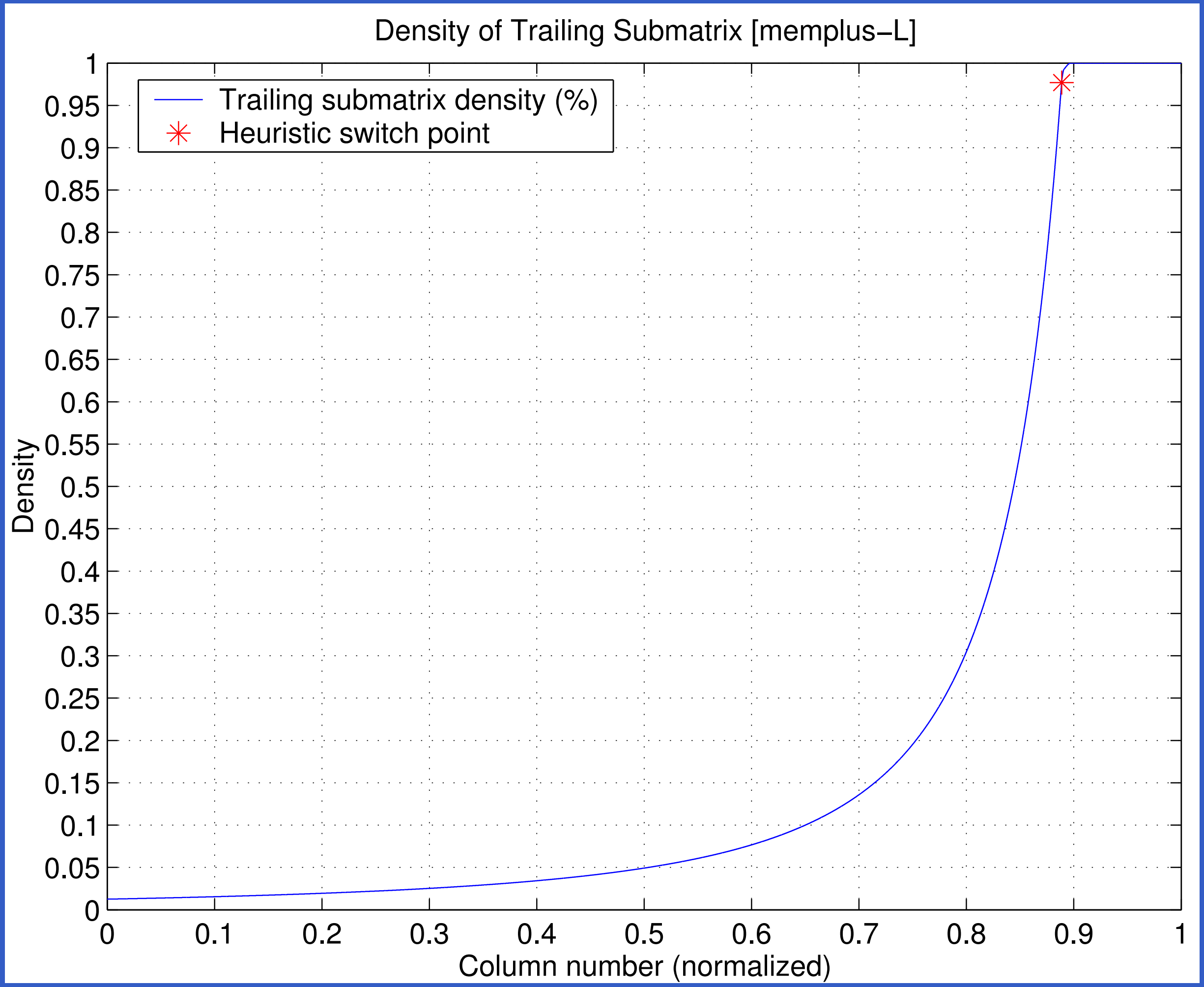




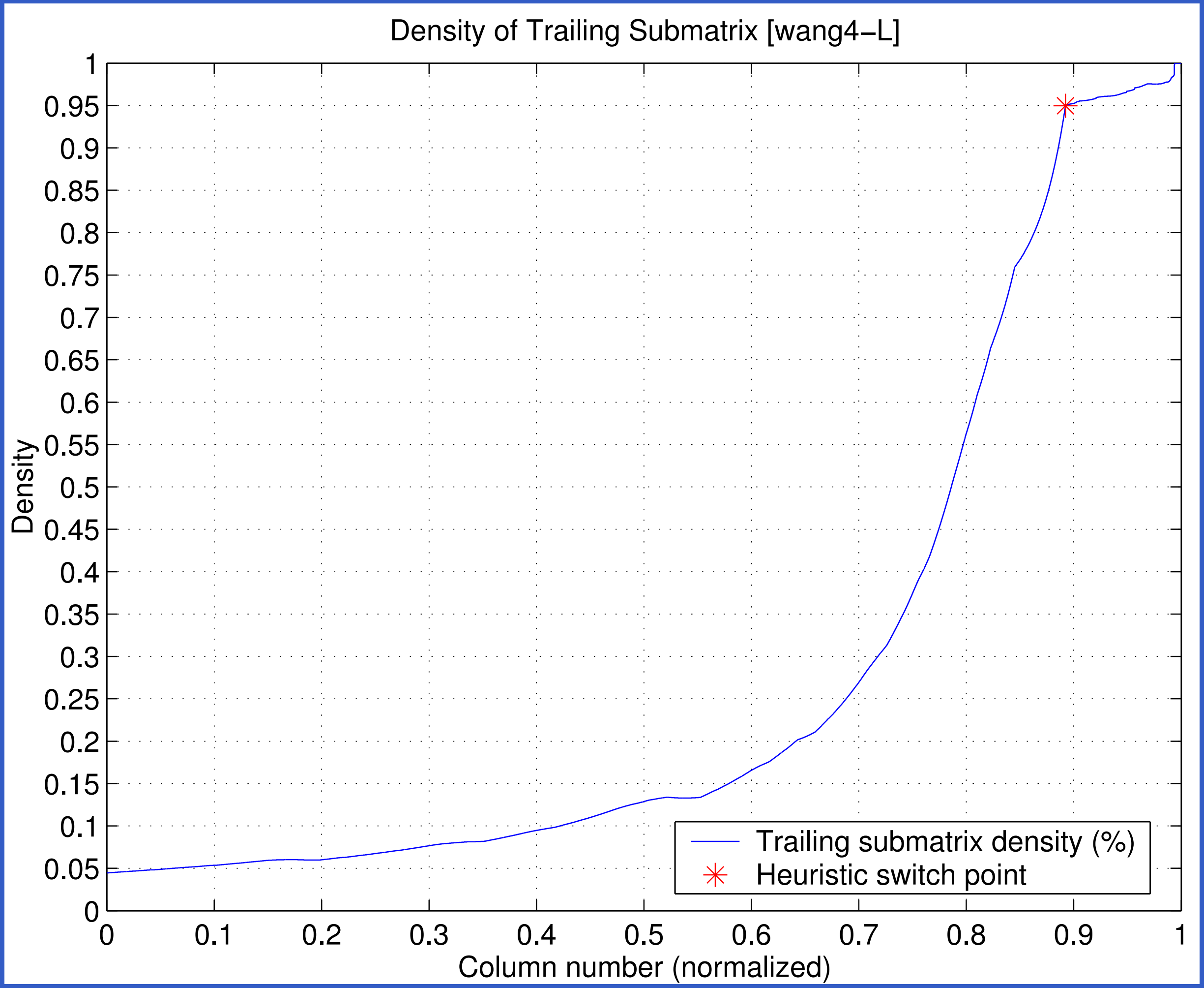
Dense Triangle Density: Dense Matrix



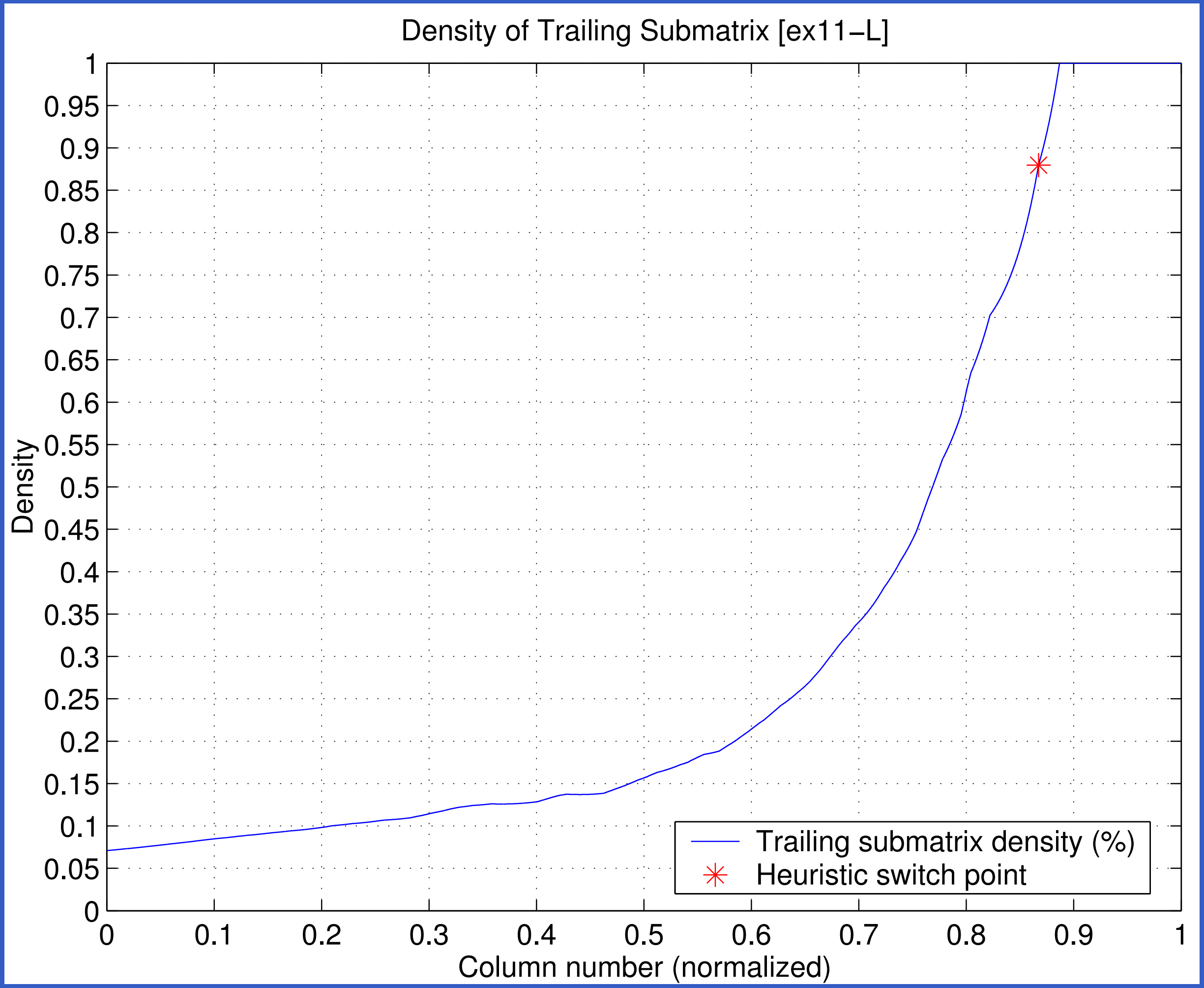
Dense Triangle Density: memplus



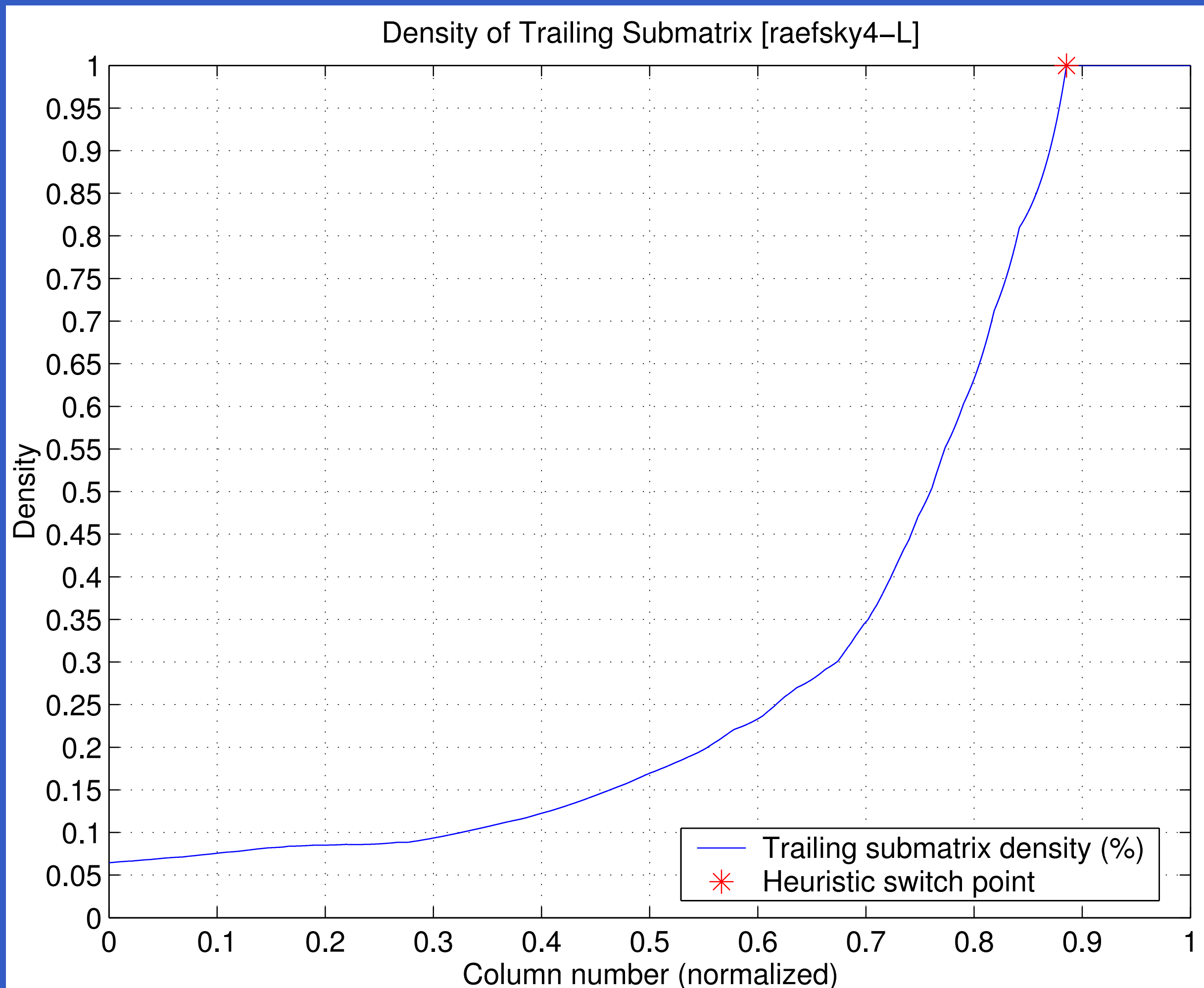
Dense Triangle Density: wang4



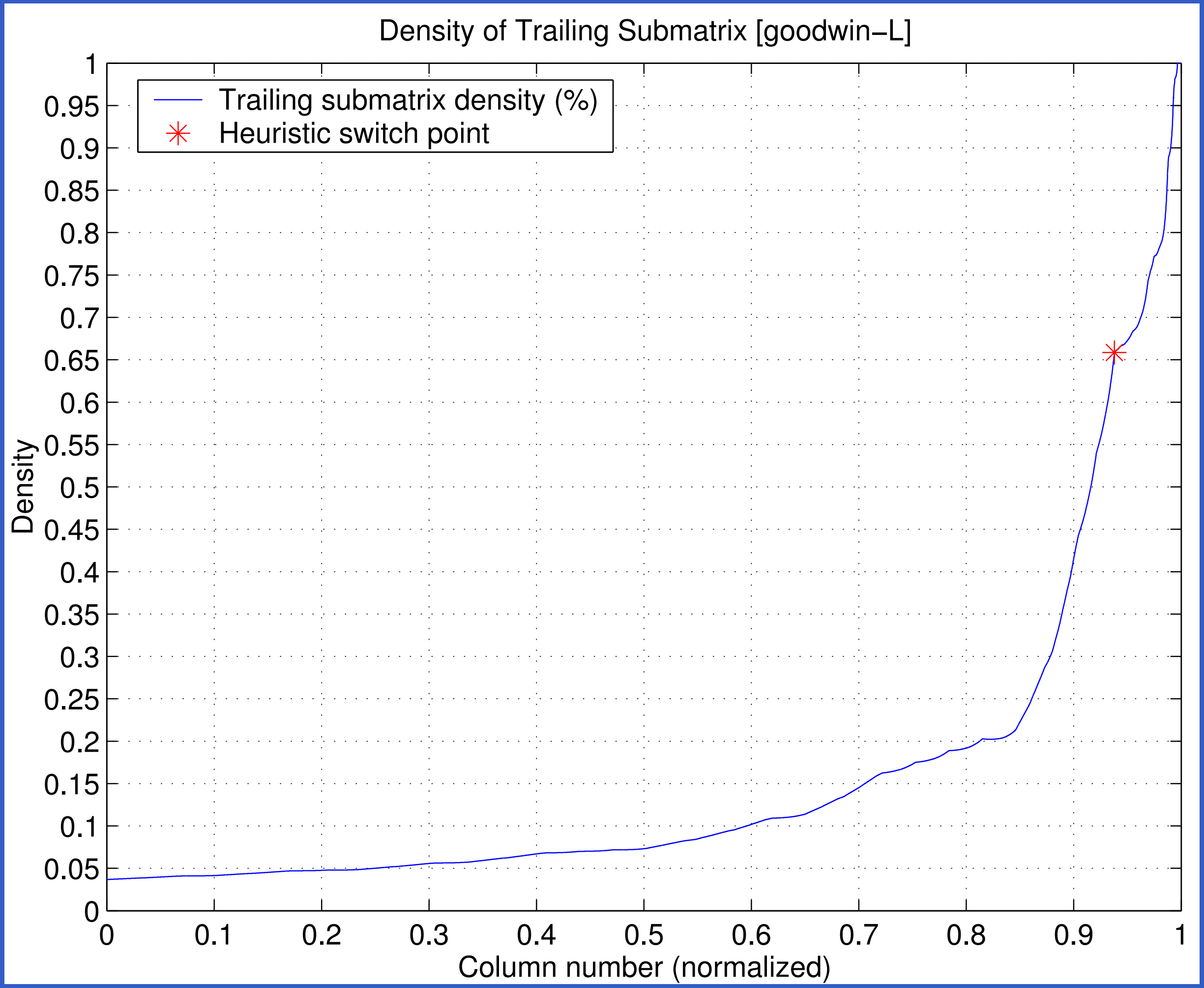
Dense Triangle Density: ex11



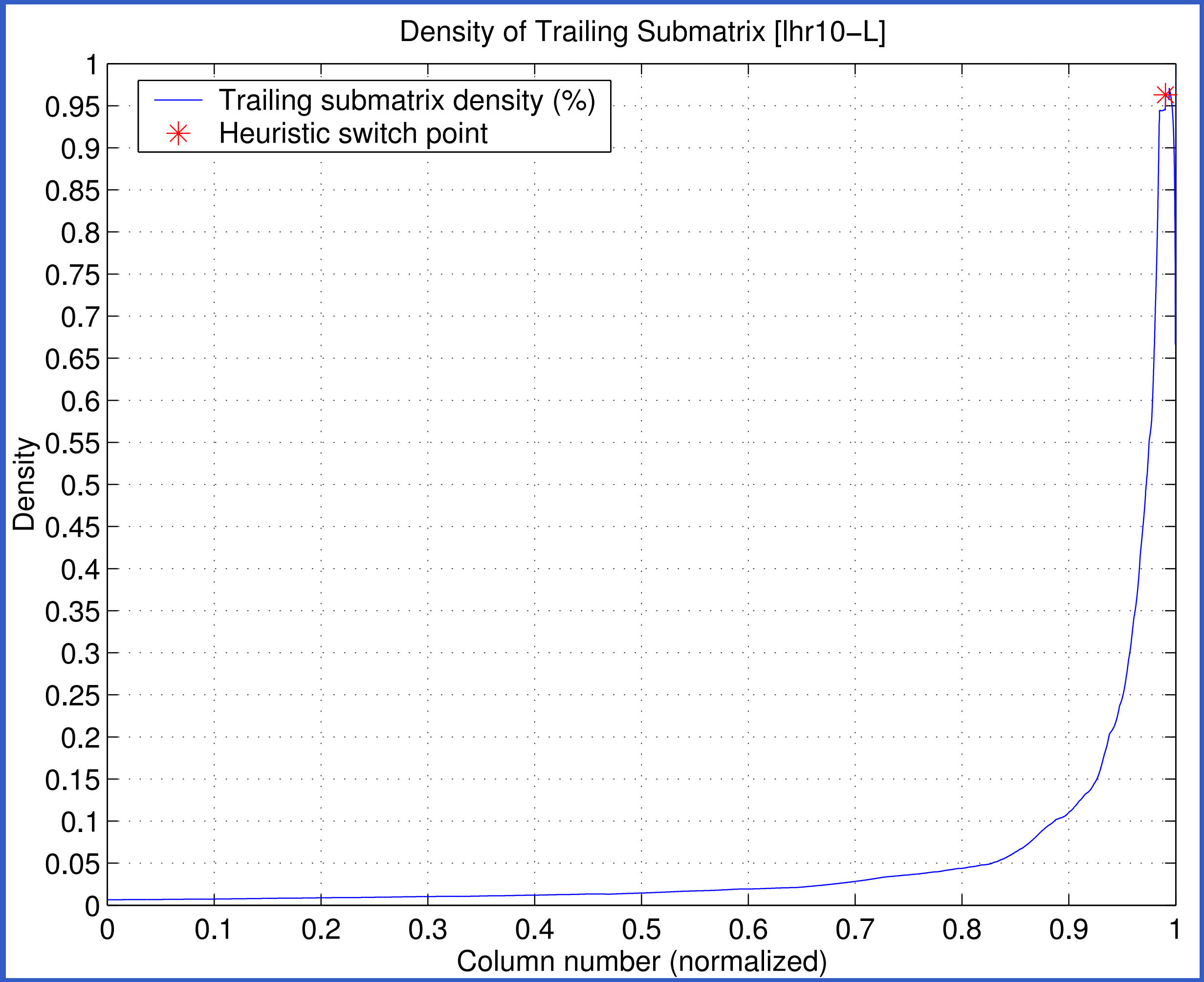
Dense Triangle Density: `raefsky4`



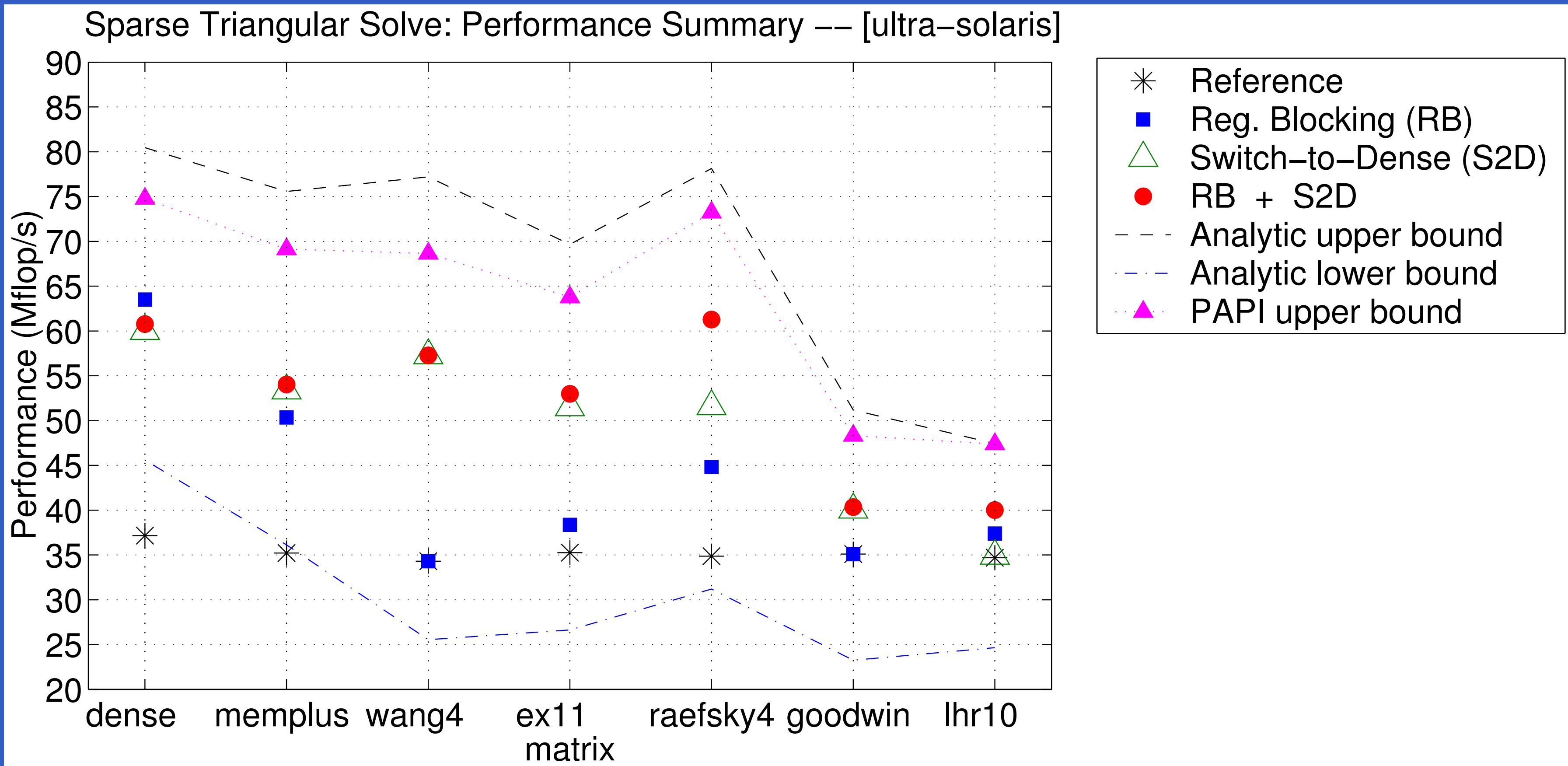
Dense Triangle Density: `goodwin`



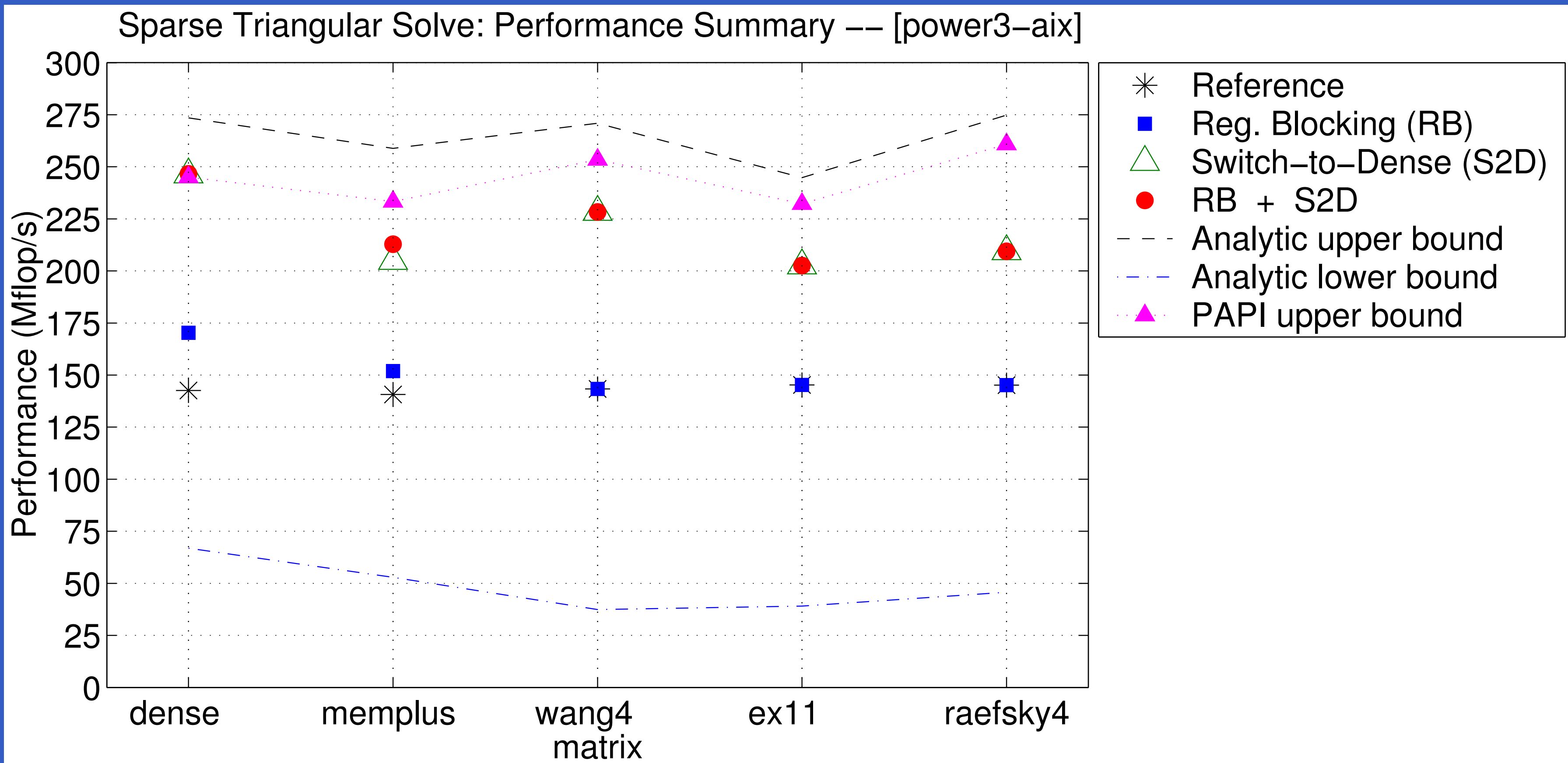
Dense Triangle Density: `lhr10`



Performance Results: Sun Ultra 2i



Performance Results: IBM Power3



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