

Appendix to “Reliable interest point detection under large illumination variations”

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Abstract

This appendix proves the illumination robustness of the proposed corner detector over the Harris corner detector.

I. HARRIS CORNER DETECTOR

The Harris corner detector [1] is based on the autocorrelation matrix of the image gradients. The autocorrelation matrix $\mathbf{A}(x, y; I)$ of an image I at a pixel (x, y) is given as follows:

$$\mathbf{A}(x, y; I) = \begin{pmatrix} \sum_{(m,n) \in N} \left(\frac{\partial}{\partial x} I(m, n) \right)^2 & \sum_{(m,n) \in N} \frac{\partial}{\partial x} I(m, n) \frac{\partial}{\partial y} I(m, n) \\ \sum_{(m,n) \in N} \frac{\partial}{\partial x} I(m, n) \frac{\partial}{\partial y} I(m, n) & \sum_{(m,n) \in N} \left(\frac{\partial}{\partial y} I(m, n) \right)^2 \end{pmatrix}, \quad (1)$$

where $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ calculates the gradients in horizontal and vertical directions, respectively; and N is a set of pixels around (x, y) . Usually a weighted sum of the gradients in N is taken using a Gaussian function centered at (x, y) to give more weight to the pixels that are close to (x, y) . The cornerness response function of the Harris corner detector is based on the determinant and trace of the autocorrelation matrix:

$$R_{harris}(x, y) = \det \mathbf{A}(x, y; I) - k(\text{trace} \mathbf{A}(x, y; I))^2, \quad (2)$$

where k is a small positive constant controlling the cornerness sensitivity of the detector. After calculating the cornerness response for all pixels, non-maxima suppression is used to get the corner points.

II. CONTRAST INVARIANT FEATURE TRANSFORM

The contrast invariant feature transform (CIFT) method improves the illumination invariance of feature detectors. The underlying idea is to stretch the image contrast as a function of intensity to span the space of possible photometric transformations. By applying a feature detector to a contrast-stretched image, the response of the feature detector under a particular illumination condition is simulated. The collection of the responses under a set of illumination conditions forms a signature for each pixel. The signatures can then be used to characterize a pixel and to find illumination robust interest points.

The contrast stretching function that we use in our experiments is the sigmoid function, which has the following form:

$$f_c(x) = \frac{1}{1 + e^{-\gamma(x-c)}}, \quad (3)$$

where x is the normalized intensity value in the range $[0, 1]$, c is the contrast center around which the contrast is stretched, and γ determines the slope of the sigmoid function.

Applying this idea to the Harris corner detector, first, the autocorrelation matrix at each pixel is found:

$$\mathbf{A}(x, y; I_c) = \begin{pmatrix} \sum_{(m,n) \in N} \left[\frac{\partial}{\partial x} I_c(m, n) \right]^2 & \sum_{(m,n) \in N} \left[\frac{\partial}{\partial x} I_c(m, n) \right] \left[\frac{\partial}{\partial y} I_c(m, n) \right] \\ \sum_{(m,n) \in N} \left[\frac{\partial}{\partial x} I_c(m, n) \right] \left[\frac{\partial}{\partial y} I_c(m, n) \right] & \sum_{(m,n) \in N} \left[\frac{\partial}{\partial y} I_c(m, n) \right]^2 \end{pmatrix}, \quad (4)$$

where $I_c(m, n) \equiv f_c(I(m, n))$ is the contrast-stretched image at contrast center c . And then, the cornerness response at the contrast center c is calculated:

$$R(x, y; c) = \det \mathbf{A}(x, y; I_c) - k(\text{trace} \mathbf{A}(x, y; I_c))^2. \quad (5)$$

Instead looking at the cornerness response at a particular c value, it makes sense to get an average response by integrating $R(x, y; c)$ over all c . Therefore, we may define the following cornerness response:

$$R_{cift}(x, y) = \int_0^1 R(x, y; c) dc. \quad (6)$$

The question is whether R_{cift} is more robust than R_{harris} against illumination changes. This is answered in the next section.

III. PROOF OF ROBUSTNESS

We incorporate the camera response function into the picture. Let $z(m, n)$ be the irradiance and $g_\alpha(\cdot)$ be the camera response function that converts the irradiance into pixel intensity:

$$I(m, n) = g_\alpha(z(m, n)). \quad (7)$$

The parameter α is the parameter associated with the illumination change. For example, in the classical formulation of the camera response function [2], $g_\alpha(z) = a_1 + a_2(\alpha z)^{a_3}$, where (a_1, a_2, a_3) are the parameters specific to the camera, and α represents gain and/or exposure time of the sensor. In the more general polynomial formulation [3], $g_\alpha(z) = \sum_{n=0}^{N-1} a_n(\alpha z)^n$.

We will now drop the spatial positions from our notations, define $q(I) \equiv \det \mathbf{A}(I) - k(\text{trace} \mathbf{A}(I))^2$, and rewrite the responses of the Harris corner detector and the CIFT method as follows:

$$R_{harris} = q(I) = q(g_\alpha(z)), \quad (8)$$

and

$$R_{cift} = \int_0^1 q(f_c(I)) dc = \int_0^1 q(f_c(g_\alpha(z))) dc. \quad (9)$$

In order to prove that R_{cift} is less sensitive to illumination changes than R_{harris} is, all we need to do is to show that $\frac{\partial R_{cift}}{\partial \alpha} \leq \frac{\partial R_{harris}}{\partial \alpha}$. Using the chain rule, the derivative of R_{harris} is

$$\frac{\partial R_{harris}}{\partial \alpha} = \frac{\partial q}{\partial I} \frac{\partial I}{\partial \alpha} = q' \frac{\partial g_{\alpha}(z)}{\partial \alpha}. \quad (10)$$

Similarly, we can write the derivative of R_{cift} as follows:

$$\frac{\partial R_{cift}}{\partial \alpha} = \frac{\partial}{\partial \alpha} \int_0^1 q(f_c(g_{\alpha}(z))) dc = \int_0^1 q' f'_c \frac{\partial g_{\alpha}(z)}{\partial \alpha} dc. \quad (11)$$

Combining these two equations, we get $\frac{\partial R_{cift}}{\partial \alpha} = \frac{\partial R_{harris}}{\partial \alpha} \int_0^1 f'_c dc$; therefore, to complete the proof it is sufficient to show that $\int_0^1 f'_c dc \leq 1$. Using the definition of the sigmoid, we obtain

$$\int_0^1 f'_c dc = \frac{1}{1 + e^{-\gamma x}} - \frac{1}{1 + e^{-\gamma(x-1)}}, \quad (12)$$

where $x \in [0, 1]$. The maximum value of this function is reached at $x = 0.5$, where its value is 1. Therefore, $\int_0^1 f'_c dc \leq 1$ for any value of x regardless of the value of γ .

QED

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