Sample Questions

1- Optical flow equation : $\underbrace{-(I - H)}_{I_t} = \underbrace{\frac{\partial I}{\partial x}}_{I_x} u + \underbrace{\frac{\partial I}{\partial y}}_{I_y} v$

$$\Rightarrow \quad -I_t = I_x u + I_y v$$

$\longrightarrow$ Assume motion vector at $(x=3, y=3)$ is equal to the one at $(x=2, y=2)$

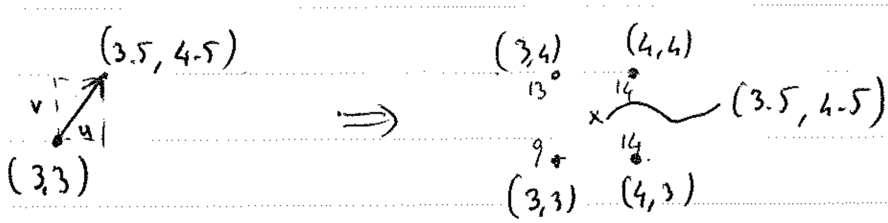
$$\Rightarrow \begin{cases} -I_t(3,3) = I_x(3,3)\,u + I_y(3,3)\,v \\ -I_t(2,2) = I_x(2,2)\,u + I_y(2,2)\,v \end{cases}$$

$$\Rightarrow \quad -\begin{bmatrix} I_t(3,3) \\ I_t(2,2) \end{bmatrix} = \begin{bmatrix} I_x(3,3) & I_y(3,3) \\ I_x(2,2) & I_y(2,2) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\Rightarrow \quad -\begin{bmatrix} 9-10 \\ 7-9 \end{bmatrix} = \begin{bmatrix} 11-9 & 9-9 \\ 8-7 & 8-7 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

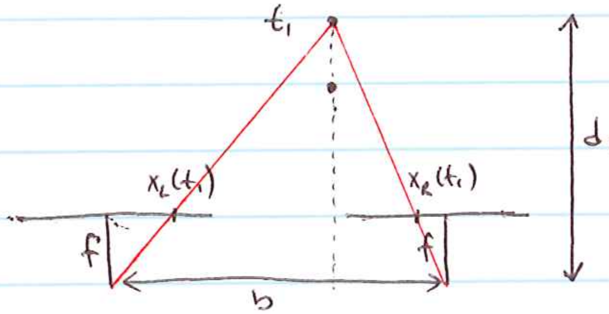
$$\Rightarrow \quad \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

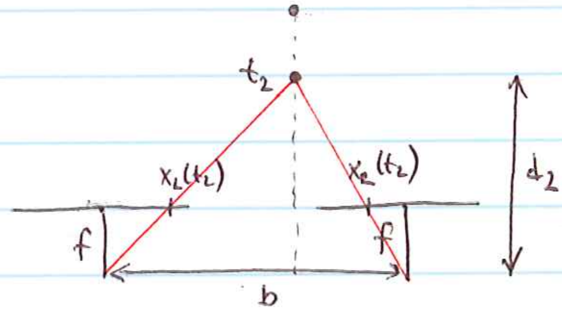
$$\Rightarrow \quad \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.5 \end{bmatrix}$$



$$I(3.5, 4.5) = \frac{13+14+9+14}{4} = \underline{12.5}$$

2 -





$$d_1 = \frac{bf}{x_L(t_1) - x_R(t_1)} \qquad d_2 = \frac{bf}{x_L(t_2) - x_R(t_2)}$$

$$v = \frac{d_1 - d_2}{t_2 - t_1} = \frac{\dfrac{bf}{x_L(t_1) - x_R(t_1)} - \dfrac{bf}{x_L(t_2) - x_R(t_2)}}{t_2 - t_1}$$

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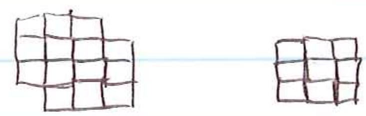
3 - $A \ominus B$ $\qquad\qquad$ $(A \ominus B) \oplus C$



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4 -

$$\underbrace{\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \\ \vdots \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \\ & & \vdots & & & \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}}_{v}$$

$$Av = b$$

$$v = (A^T A)^{-1} A^T b$$

Perspective model

$$x' = \frac{a_1 x + a_2 y + a_3}{a_7 x + a_8 y + 1} \Rightarrow a_7 x' x + a_8 x' y + x' = a_1 x + a_2 y + a_3$$

$$y' = \frac{a_4 x + a_5 y + a_6}{a_7 x + a_8 y + 1} \Rightarrow a_7 y' x + a_8 y' y + y' = a_4 x + a_5 y + a_6$$

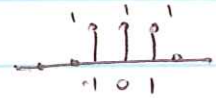
$$\Longrightarrow \begin{aligned} x' &= a_1 x + a_2 y + a_3 - a_7 x' x - a_8 x' y \\ y' &= a_4 x + a_5 y + a_6 - a_7 y' x - a_8 y' y \end{aligned}$$

$$\Rightarrow \underbrace{\begin{bmatrix} x_1' \\ y_1' \\ \vdots \\ \\ \\ \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1' x_1 & -x_1' y_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_1' x_1 & -y_1' y_1 \\ & & & & \vdots & & & \\ & & & & & & & \\ & & & & & & & \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix}}_{v}$$

At least three more correspondences, that is, 6 more lines (equations) are needed.

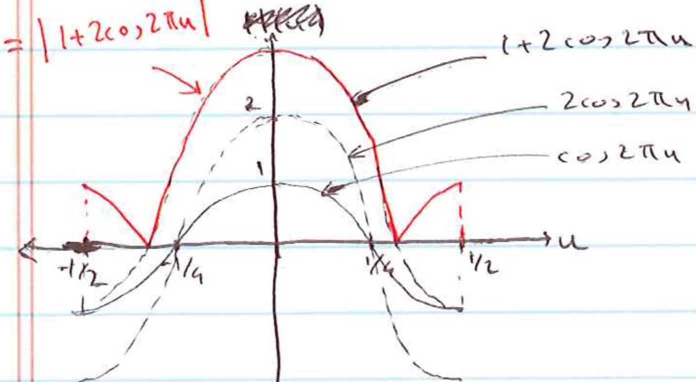
$$v = (A^T A)^{-1} A^T b$$

5 – $h[n] = \begin{cases} 1 & \text{for } n = -1, 0, 1 \\ 0 & \text{o/w} \end{cases}$

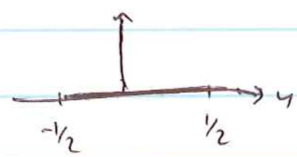


$$H(u) = \sum_{n=-\infty}^{\infty} h[n]\, e^{-j2\pi u n} = 1 \times e^{j2\pi u 1} + 1 \times e^{-j2\pi u 0} + 1 \times e^{-j2\pi u 1}$$

$$= e^{j2\pi u} + 1 + e^{-j2\pi u} = 1 + \underbrace{e^{j2\pi u} + e^{-j2\pi u}}_{2\cos 2\pi u} = 1 + 2\cos 2\pi u$$



Phase$\{H(u)\} = 0$



$\rightarrow \quad \displaystyle\sum_{n=-\infty}^{\infty} |h[n]|^2 = 1^2 + 1^2 + 1^2 = 3$ //

$$\rightarrow \quad \int_{-1/2}^{1/2} |H(u)|^2\, du = \int_{-1/2}^{1/2} (1 + 2\cos 2\pi u)^2\, du = \int_{-1/2}^{1/2} (1 + 4\cos 2\pi u + 4\cos^2 2\pi u)\, du$$

$$= \int_{-1/2}^{1/2} (1 + 4\cos 2\pi u + 2 + 2\cos 4\pi u)\, du$$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$$= \left[3u + \frac{4}{2\pi}\sin 2\pi u + \frac{2}{4\pi}\sin 4\pi u\right]_{-1/2}^{1/2} = 3u\Big]_{-1/2}^{1/2} = 3$$ //

6-

$$G_1 = H_1 F + N_1$$
$$G_2 = H_2 F + N_2$$

$$\text{Cost} = a_1 |G_1 - H_1 F|^2 + a_2 |G_2 - H_2 F|^2$$
$$= a_1 (G_1 - H_1 F)^* (G_1 - H_1 F) + a_2 (G_2 - H_2 F)^* (G_2 - H_2 F)$$

$$\frac{\partial \text{Cost}}{\partial F} = a_1 (G_1^* - H_1^* F^*)(-H_1) + a_2 (G_2^* - H_2^* F^*)(-H_2)$$

$$= -a_1 G_1^* H_1 + a_1 H_1^* H_1 F^* - a_2 G_2^* H_2 + a_2 H_2^* H_2 F^*$$

Set $\frac{\partial \text{Cost}}{\partial F} = 0$ to find the optimum values:

$$\Rightarrow F^* (a_1 |H_1|^2 + a_2 |H_2|^2) = a_1 G_1^* H_1 + a_2 G_2^* H_2$$

$$\Rightarrow \boxed{F = \frac{a_1 H_1^* G_1 + a_2 H_2^* G_2}{a_1 |H_1|^2 + a_2 |H_2|^2}}$$