Error

Error is the difference between an ideal (or correct) value and an actual value.

- Several different types of error can be measured.
- An error type can be expressed in several ways.

Expression of Error

Notation

$I$ denotes an ideal value.

$A$ denotes an actual value.

Absolute error defined $|I - A|$.

Percent error defined $100\frac{|I - A|}{I}$ for $I \neq 0$.

Consider a transducer designed to measure process variables in the range $I \in [x_{\min}, x_{\max}]$.

Percent-full-scale error defined $100\frac{|I - A|}{x_{\max}}$ for $x_{\max} \neq 0$.

Types of Error

- Model Error.
  Error in transducer model, $H_t$.

- Repeatability Error.
  Transducer change from occasion to occasion.

- Stability Error.
  Transducer change during use.

- Calibration Error.
  Difference between two transducers of same kind.

Model Error

Let $y = H_t(x)$ denote a transducer output, response, and process variable.

The accuracy of $H_t(x)$ depends upon how well the transducer is understood and how complex a transfer function can be tolerated.

For example, the following are all for the same transducer:

Okay: $H_{t1}(x) = R_o(1 + ax)$.

Good: $H_{t2}(x) = R_o(1 + ax + bx^2)$.

Better: $H_{t3}(x) = R_o(1 + ax + bx^2 + cx^3)$.

Best: $H_{t4}(0^\circ C) = 100 \Omega$, $H_{t4}(0.01^\circ C) = 100.35 \Omega$, $\ldots$ (Called a lookup table.)

Model error quantifies the accuracy of the transfer function.

Definition of Model Error Quantities

Test conditions: a single measurement. Let $H_t(x)$ denote the transducer response, $x$ denote the process-variable value, and $y$ the quantity measured at the transducer outputs.

Then: Ideal: $I = x$, Actual: $A = H_t^{-1}(y)$.
Model Error Example

What is the absolute model error of a transducer having response \( H_t(x) = (10x^2 - 5) \) V under test conditions, with process variable \( x = 2.130 \) and measured transducer output \( y = 34.90 \) V.

The ideal quantity is \( I = 2.130 \).

\[
H_t^{-1}(y) = \sqrt{\frac{y}{10} + 5}.
\]

Based on the transducer \( A = H_t^{-1}(34.90) = 1.998 \).

The absolute error is then, 0.1325.

Repeatability

Measures how well a transducer performs over time.

Definition of Repeatability Error Quantities

Test conditions:

Let \( H_t(x) \) denote the transducer response.

Let \( x(t) \) denote the value of the process variable at time \( t \).

Two measurements are made, at times \( t_1 \) and \( t_2 \), \( t_1 < t_2 \).

The test is set up so that \( x(t_1) = x(t_2) = x \) and \( x(t_{1,5}) \neq x \) for some \( t_1 < t_{1,5} < t_2 \).

Let \( y_1 \) and \( y_2 \) denote the quantities read at the transducer outputs at times \( t_1 \) and \( t_2 \).

Then: Ideal: \( I = H_t^{-1}(y_1) \). Actual: \( A = H_t^{-1}(y_2) \).

Stability

Measures how well the a transducer measures a steady quantity.

Definition of Stability Error Quantities

Test conditions:

Let \( H_t(x) \) denote the transducer response.

Let \( x(t) \) denote the value of the process variable at time \( t \).

Two measurements are made, at times \( t_1 \) and \( t_2 \), \( t_1 < t_2 \).

The test is set up so that \( x(t_1) = x(t_2) = x \) for all \( t_1 < t_{1,5} < t_2 \).

Let \( y_1 \) and \( y_2 \) denote the quantities read at the transducer outputs at \( t_1 \) and \( t_2 \).

Then: Ideal: \( I = H_t^{-1}(y_1) \). Actual: \( A = H_t^{-1}(y_2) \).

Calibration

Measures how well two transducers of the same type compare.

Definition of Calibration Error Quantities

Test conditions:

Let \( H_t(x) \) denote the transducer response and \( x \) denote the value of the process variable.

A measurement is made with each transducer.

Let \( y_1 \) and \( y_2 \) be the quantities read at the transducers’ outputs.

Then: Ideal: \( I = H_t^{-1}(y_1) \). Actual: \( A = H_t^{-1}(y_2) \).
Example

A type of integrated temperature sensor has a response of $H_t(x) = 7.2 \frac{\mu A}{K}$. Tests were performed on two such sensors by exposing the sensors to a known temperature, $x$, and measuring their response, $y$, as follows:

At time $t_1$ sensor A exposed to $x = 295$ K; output $y = 2050 \mu A$.
At time $t_2$ sensor A exposed to $x = 300$ K; output $y = 2085 \mu A$.
At time $t_3$ sensor A exposed to $x = 295$ K; output $y = 2052 \mu A$.
At time $t_4$ sensor A exposed to $x = 295$ K; output $y = 2053 \mu A$.
At time $t_5$ sensor B exposed to $x = 295$ K; output $y = 2040 \mu A$.

Temperature is held constant from $t_3$ to $t_5$. Find model error, repeatability error, stability error, and calibration error.

Inverted Model Function

$x = H_{t_1}^{-1}(y) = 7.2 \frac{K}{\mu A}$. $y = \frac{7.2}{K}$. $A = \frac{1}{7.2}$.

Model Error

Use measurement at $t_1$.

$I = 295.0$ K and $A = H_{t_1}^{-1}(2050 \mu A) = 292.9$ K.

Percent model error: $\frac{|295.0 - 292.9|}{295.0} = 0.71\%$.

Could have used any time to compute model error.

Repeatability Error

Use measurements at $t_1$ and $t_3$ (since temperature different at $t_2$).

$I = H_{t_1}^{-1}(y(t_1)) = H_{t_1}^{-1}(2050 \mu A) = 292.9$ K.

$A = H_{t_1}^{-1}(y(t_3)) = H_{t_1}^{-1}(2052 \mu A) = 293.1$ K.

Percent repeatability error: $\frac{|292.9 - 293.1|}{292.9} = 0.06828\%$.

Note, actual and ideal quantities could be reversed in this example. Also possible to use $t_1$ and $t_4$.

Stability Error

Use measurements at $t_3$ and $t_4$ (since temperature held constant in this time range).

$I = H_{t_1}^{-1}(y(t_3)) = H_{t_1}^{-1}(2052 \mu A) = 293.1$ K.

$A = H_{t_1}^{-1}(y(t_4)) = H_{t_1}^{-1}(2053 \mu A) = 293.3$ K.

Percent stability error: $\frac{|293.1 - 293.3|}{293.1} = 0.06824\%$. 
Calibration Error

Use measurements at $t_4$ and $t_5$.

$I = H^{-1}_1(g(t_4)) = H^{-1}_1(2053 \mu A) = 293.3 K$.

$A = H^{-1}_1(g(t_5)) = H^{-1}_1(2040 \mu A) = 291.4 K$.

Percent calibration error: \[
\frac{|293.3 K - 291.4 K|}{293.3 K} = 0.6478\%.
\]