Temperature

Definition: The translational (e.g., wiggling around) energy of particles in a system.

No practical way to measure velocity of every particle in most systems of interest.

Instead, temperature scales are defined.

There are two types:

- The thermodynamic temperature scale. "Really" measures temperature.
- Practical temperature scales. Approximations of thermodynamic scale.

Much easier to measure temperature on a practical scale. For temperatures of interest, differences are very small.

Kelvin’s Thermodynamic Temperature Scale

Due to William Thomson, a.k.a., Lord Kelvin (1824-1907).

Start with a precise temperature that can easily be reproduced. The triple point of water, $T_{tr}$, is used.

Confine an ideal gas in a container of fixed volume, $S$.

Bring the gas to temperature $T_{tr}$.

Call the pressure of this gas $P_{tr}$.

By definition (of the Kelvin scale) this temperature is $T_{tr} = 273.16 \text{ K}$.

The ideal gas law: $P_S = nR_T$.

Substituting, $P_{tr}S = nR_{273.16 \text{ K}}$. Then $nR = \frac{P_{tr}S}{273.16 \text{ K}}$.

Bring the same system to another temperature, $T$.

Call the pressure at this temperature, $P$.

Solving for $T$ in the gas law:

$$T = \frac{PS}{nR} = \frac{P_{tr}S}{273.16 \text{ K}} \frac{P}{P_{tr}}$$

Practical Temperature Scales

Designed to be easy (relatively) to measure.

Scales are revised every few decades.

Latest revision in 1990, called ITS-90. (International Temperature Scale.)

Older scale (1968), IPTS-68. (International Practical Temperature Scale)

Difference between ITS-90 and IPTS-68 is as large as 0.4°C. At human-tolerable temperatures, difference is in hundreths of a degree.

All practical scales are identical at the triple point of water.

For ITS-90:

- (65 K, 5.0 K) Vapor-pressure relation between two isotopes of helium.
- (3.0 K, 24.5561 K) Helium fixed-volume thermometer. (Like thermometer used in thermodynamic scale, . . . except helium replaces the ideal gas.)
- > 1234.93 K: Based on radiated light.
Temperature Transducers

Basic Types

- **Thermistor.**
  Block of semiconductor material.
  Resistance is a function of temperature.
- **Resistance Temperature Device (RTD)**
  Strip of metal.
  Resistance is a function of temperature.
- **Thermocouple.**
  Potential across two metals is a function of temperature.
- **Diode.**
  Forward-bias voltage is a function of temperature. (Not covered.)

Integrated Temperature Sensors

- Transducer and factory-calibrated conditioning circuit ... combined in a single package.
- Usually available as current or voltage sources.
- Current or voltage is a convenient, linear function of temperature.

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Thermistor

**Name:** Thermal resistor.

**Symbols:** (Both are used.)

**Temperature range:** about \(-100^\circ\text{C} \text{ to } 200^\circ\text{C} \). (Relatively narrow.)

**Construction:**
- block of semiconductor material (without junction).

**Principle of Operation**

As with all semiconductors, ... 

- ... electron energy levels divided into two bands, ... 
  - ... valence and conduction.

- Electrons in conduction band participate in current flow.
- Electrons in valence band do not.¹

**Desirable Characteristics**

- Sensitive. (Small change in temperature yields an easily readable change in resistance.)
- Can be made very small. (Small devices react to temperature changes quickly.)
- High resistance. (Easier to design conditioning circuit.)

**Undesirable Characteristics**

- Delicate. Can be damaged (de-calibrated) by excessive heat.
- There are many non-standard types.

**Transducer Model Functions**

- All functions will be approximations.

**Very good, the Steinhart-Hart Equation:**

\[
H_{t1}^{-1}(y) = \left( \frac{1}{A + B \ln y + C \ln^3 y} \right)^{-1},
\]

where \(A, B,\) and \(C\) are experimentally determined constants.

**Good:** \(H_{t2}(x) = R_0 e^{\beta x}\).

Later, a linear function will be derived.

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1. Actually they do, but that’s a hole other story.

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**Thermistor Sample Problem**

Convert process variable \(x \in [-10^\circ\text{C}, 50^\circ\text{C}]\), the temperature in room 102 EE Building into \(H(x) = x \frac{1}{K}\), a floating-point number. The number should have a precision of 0.05. Use a thermistor and the function \(H_{t2}(x) = R_0 e^{\beta x}\) with \(\beta = 3000\text{K}\) and \(R_0 = 0.059\Omega\).

**Solution Plan:**

- Choose ADC.
  - Based on ADC input voltage, design conditioning circuit.
  - Based on ADC precision (bits), write interface routine.

**ADC Choice**

Use ADC with function \(H_{ADC}(V,V,B)(y)\) ... 

- ... value of \(b\) chosen later.
Conditioning Circuit

Input is a resistance (from thermistor), output is voltage.

Input range to ADC is 0 to 5 V, therefore:

\[ 0 \leq H_t(x_2) \leq 5 \] for \(-10^\circ C \leq x \leq 50^\circ C\)

Choose conditioning circuit based on this constraint.

Conditioning circuit will not linearize \(x\).

(This would be very difficult using analog circuits.)

Thermistor \(y = H_t(x)\) is monotonic with temperature.

In this case, when \(x\) increases \(y\) always decreases.

Therefore, conditioning circuit must convert either:

\[ H_{t1}(-10^\circ C) = 0 \text{ V and } H_{t2}(50^\circ C) = 5 \text{ V} \]

or

\[ H_{t1}(-10^\circ C) = 5 \text{ V and } H_{t2}(50^\circ C) = 0 \text{ V} \]

ADC Output, ADC Precision Choice

Problem specified that \(H(x)\) should have a 0.05 precision.

ADC Output:

\[ H_{ADC(VAC)}(H_t(x_2)) = z = \frac{1}{V_{ADC}}(2^b - 1)A_3(R_{max}/2 - O_3) \]

To determine precision evaluate at \(x_1 = 323.10\) K and \(x_2 = 323.15\) K.

Difference should be no less than one.

\[ H_{ADC(VAC)}(H_t(x_1)) - H_{ADC(VAC)}(H_t(x_2)) \geq 1 \]

\[ \frac{1}{V_{ADC}}(2^b - 1)A_3(R_{max}/2 - R_{min}/2) \geq 1 \]

Solving for \(b\) yields:

\[ b \geq \left\lfloor \log_2 \left( \frac{V_{ADC}}{A_3(R_{max}/2 - R_{min}/2)} + 1 \right) \right\rfloor = 13. \]

Interface Routine

\[ H_{ADC(VAC)}(H_t(x_2)) = z = \frac{1}{V_{ADC}}(2^b - 1)A_3(R_{max}/2 - O_3) \]

Solving for \(x\) yields:

\[ x = \beta \left( \ln \left( \frac{O_3}{B_0} + \frac{1}{A_3B_0}(2^b - 1) \right) \right)^{-1} \]

Substituting values:

\[ \text{tee} = 3000.0 / (\log(10760.3 + 9.5949 * \text{raw})) \]

where \(\text{raw}\) is the value read from the ADC output.
Linear Thermistor Model

Linear transducer functions are preferred.

Especially useful when there is no computer processing.

Thermistor response is close to linear over small temperature ranges.

But non-linear over wide temperature ranges.

A linear thermistor function will be derived.

Plan:

Call $T_d$ the “middle” temperature.

(Center of range of temperatures to measure.)

Goal: derive function in form $H_{A}(x) = R_d(1 + a\Delta x)$ .

... where $R_d$ and $a$ are constants to be determined ...

... and $\Delta x = x - T_d$

Temporarily set $H_{A}(x) = mx + b$, the equation of a straight line.

Let $m = \left(\frac{d}{dx}H_{A}(x)\right)_{x=T_d}$.

Solve for $b$ in $mT_d + b = H_{A}(T_d)$.

Transform $mx + b$ into $H_{A}(1 + a\Delta x)$.

Then:

$$R_d = H_{A}(T_d) = R_d\left(1 + \frac{\Delta x}{T_d}\right)$$

and

$$a = -\frac{\beta}{T_d}$$

Note: derivation can also be done using a more accurate model than $H_{A}(x)$.

Passive Conditioning Circuit

Idea:

Place thermistor in a resistor network to achieve some linearity.

A simple but effective example appears below.

For convenience, combination will be treated as a transducer.

![Diagram of Passive Conditioning Circuit]

Transfer function: $H_{A}(x) = \frac{R_d}{2}\left(1 + \frac{\alpha}{2}\Delta x\right)$ .

where $R_d$ is resistance at center of range,

$\Delta x = x - T_d$.

$T_d$ is the temperature at the center of range,

and $\alpha = \left(\frac{d}{dx}H_{A}(x)\right)_{x=T_d}$.

How much more accurate is this?

Thermistor Linearization Sample Problem

Compute the model error of thermistor functions $H_{A}$ and $H_{A}$ at temperatures 250 K, 270 K, and 290 K for a thermistor and a thermistor with a shunt resistor (the passive conditioning circuit just presented) designed for temperature range [250 K, 290 K].

Base the error on the following measurements:

$R_d(250 \text{K}) = 9603 \Omega$,

$R_d(270 \text{K}) = 3948 \Omega$,

$R_d(290 \text{K}) = 1835 \Omega$,

where $R_d(T)$ is the measured resistance of the thermistor at temperature $T$.

Thermistor model functions.

$H_{A}(x) = R_d(1 + a(x - T_d))$ and $H_{A}(x) = \frac{R_d}{2}\left(1 + \frac{\alpha}{2}(x - T_d)\right)$.

where $T_d = 270 \text{K}$, $\alpha = -\frac{\beta}{T_d} = -0.04115$ and $R_d = 3948 \Omega$.

The inverse of functions are:

$$H_{A}^{-1}(R_d) = \frac{1}{\alpha}\left(\frac{R_d}{R_d} - 1\right) + T_d$$

and

$$H_{A}^{-1}(R_d) = \frac{2}{\alpha}\left(\frac{2}{R_d} - 1\right) + T_d$$.

<table>
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</tr>
<tr>
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<td>1835</td>
<td>1253</td>
<td>2.42%</td>
<td>283.0</td>
<td>0.78%</td>
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</table>
Plotted below are the actual temperature and the temperature computed using the two thermistor-model functions.

There are more elaborate networks which can be used to linearize thermistor response. These will not be covered.

Resistance Temperature Device (RTD)

Symbol:

Temperature range: about $-220 \, ^\circ C$ to $750 \, ^\circ C$ (Platinum).

Construction:
Transducer is a metal, usually platinum.
Metal is wound into a long coil . . .
. . . or printed on a ceramic substrate in a serpentine pattern.
Resistance change of metals discovered in 1821 by Sir Humphrey Davy.
Use of platinum for temperature measurement . . .
. . . suggested by Sir William Siemens in 1871.

Principle of Operation
In normal operation a current is flowing through RTD.
As the temperature increases, . . .
. . . electrons collide more frequently with metal atoms . . .
. . . increasing resistance.

Desirable Characteristics
- Accurate. (An RTD used to define part of ITS-90.)
- Stable.
- Wide temperature range.
- Reasonably linear.
- Available in standard types.

Undesirable Characteristics
- Low sensitivity.
  (Small resistance change with temperature.)
- Expensive.

RTD Model Function
For a properly constructed transducer . . .
. . . the following function is exact for ITS-90:

$$H_{t0}(x) = R_0 \left( C_0 + \sum_{i=1}^{9} C_i \left( \frac{x}{K - 754.15} \right)^i \right).$$

for $x \in (0 \, ^\circ C, 961.78 \, ^\circ C)$, where $R_0 = H_{t0}(273.16 \, K)$ and the $C_i$ are constants defined in the ITS-90 standard.

This function is accurate (but not exact) and easier to use:

$$H_{t1}(x) = R_0 \left( 1 + \alpha_1 x + \alpha_2 x^2 \right).$$

Platinum RTDs are usually made so that $R_0 = 100 \, \Omega$.
For platinum, $\alpha_1 = 0.00398/\, ^\circ C$ and $\alpha_2 = -5.84 \times 10^{-7}/\, ^\circ C^2$. 
Three-Wire Configuration

Problem with RTDs: lead resistance is significant.

- Lead resistance may change with temperature.
- Not necessarily the temperature being measured!

Leads can be shortened or lengthened, changing resistance.

Solution: three-wire RTD.

Three-Wire RTD Use

When used in a Wheatstone bridge, ... unwanted resistances have little effect on output.

\[ v_o = \frac{4}{R_w} \left( \frac{R_0 H_t(x)}{R_0 H_t(x) + 2 R_w + 2 R_0} \right) = \frac{4 v_E}{H_t'(x)}. \]

\( R_w \) is resistance of wire. Output, \( v_o \), should not be a function of this.

Re-write RTD function as \( H_t(x) = R_0 + R_0 H_t'(x) \),

where \( H_t(x) = \alpha_1 x + \alpha_2 x^2 \).

Thermocouples

Symbol: ❒

Temperature range: about \([-270^\circ C; 1820^\circ C]\).

Construction:
- Two metals joined (e.g., welded together).
- Transducer is junction (weld).
- Leads, connecting to metals, specially constructed.

Principle of Operation

When dissimilar metals joined, an EMF develops across the junction.

(As in semiconductor PN junctions.)

Strength of EMF is a function of metals used and junction temperature.

Temperature is determined by measuring voltage.

Measuring this voltage is not as easy as one might think.

History

Effect of EMF discovered in 1821 by Thomas Seebeck.

Desirable Characteristics

- Very wide temperature range: freezer to furnace.
- Rugged.
- Accurate.
- Highly standardized.

Connectors and color of cables are all part of the standard.

Tables of thermocouple voltages published by National Institute of Standards and Technology (NIST).

No self-heating.

Conditioning circuit does not warm transducer.

Undesirable Characteristics

- Difficult to measure voltage.
- Non-linear.

Transducer Model Functions

Best: \( H_{t1}(x) \), thermocouple tables published by NIST.

Over a narrow temperature range:

\( H_{t3}(x) = v_M(1 + \alpha(x - T_M)) \ldots \)

\( \ldots \) where \( \alpha \) is a constant called \ldots

\ldots the Seebeck Coefficient.

In most cases a lookup table, \( H_{t1}(x) \), would be used.
Seebeck Voltage

Consider two joined metals:

\[ v_{AB}(x) \]

Potential developed, \( v_{AB}(x) \), a function of metals and temperature.

**Inserted Metals**

Consider two junctions in series, junction AX at \( x_1 \) and junction XB at \( x_2 \):

\[ v_{AX}(x_1) \]
\[ v_{XB}(x_2) \]

\[ v = v_{AX}(x_1) + v_{XB}(x_2). \]

If \( x_1 = x_2 = x \) then

\[ v = v_{AX}(x) + v_{XB}(x) = v_{AB}(x). \]

This is referred to as the law of inserted metals.

Used to show, among other things, that thermocouple junctions can be welded.

Measurement of Seebeck Voltage

Consider the following setup:

Note that both voltmeter connections are at same temperature.

Schematically,

\[ v_M = v_{CA}(T_r) + v_{AB}(x) + v_{BC}(T_r) \]

\[ = v_{AB}(x) + v_{TA}(T_r) \]

\[ = v_{AB}(x) - v_{TB}(T_r) \]

To determine \( x \) must know \( T_r \).

Note that \( v_M \) is not a function of metal C, making life easier for us.

**Isothermal Block**

Block upon which connections to thermocouple leads are made.

All parts of block are kept at the same temperature.

Called the reference temperature, and denoted \( T_r \).

Either \( \ldots \)

\( \ldots \) block is maintained at a known temperature.

(E.g., placed in an ice bucket.) \( \ldots \)

\( \ldots \) or block also includes a temperature sensor.

Either way, \( T_r \) is treated as part of the conditioning circuit.

Using isothermal block, \( v_{AB}(T) \) can be determined.

**Standard Thermocouple Tables**

Published by NIST.

Some standard thermocouples:

- **Type J**: Iron vs. Copper-Nickel.
  - Temperature range: \([-210°C, 760°C]\).
- **Type K**: Nickel-Chromium vs. Nickel-Aluminum.
  - Temperature range: \([-270°C, 1372°C]\).
- **Type R**: Platinum-13% Rhodium vs. Platinum.
  - Temperature range: \([0°C, 1767°C]\).

Thermocouple Table Entries

Standard thermocouple tables give:

\[ v = H_{XY}(x) = v_{AB}(x) - v_{AB}(0°C) \]

and

\[ x = H_{XY}^{-1}(v), \] where XY is the type of thermocouple.

Temperatures in tables used for class are on the IPTS-68 scale.

Example:

A voltage of 6.86 mV is measured at an isothermal block connected to a Type-R thermocouple. The block is at 0°C. What is the thermocouple temperature?

According to the table, \( H_{TypeR}(710°C) = 6.860 \) mV.

So, temperature is 710°C.
When Isothermal Block is not at 0°C:

Recall, \( H_{XY}(x) = v_{XY}(x) - v_{XY}(0°C) \).

Consider a measurement where \( T_r \neq 0°C \).

Then we need: \( v_{XY}(x) - v_{XY}(T_r) \).

This is equal to \( H_{XY}(x) - H_{XY}(T_r) \).

Example:

A voltage of 6.860 mV is measured at an isothermal block connected to a Type-R thermocouple. The block is at 23°C. What is the thermocouple temperature?

By the Type-R thermocouple table:

\( H_{TypeR}(710°C) = 6.860 \text{ mV} \) and \( H_{TypeR}(23°C) = 0.129 \text{ mV} \).

Measured voltage is \( v_{TypeR}(x) - v_{TypeR}(0°C) \).

Subtract \( v_{TypeR}(0°C) \) from both sides and solve for \( v_{TypeR}(x) - v_{TypeR}(0°C) \).

Substituting values, \( v_{TypeR}(x) - v_{TypeR}(0°C) = 6.989 \text{ mV} \).

Based on table, \( x = 721°C \).

Ice-Bath Circuits

Compensate for temperature of isothermal block.

Other Names:

- Electronic ice point.
- Hardware compensation.

Details

Consider an isothermal block ... with a built-in temperature transducer.

A circuit which converts the voltage at the thermocouple leads ... from \( v_{XY}(x) - v_{XY}(T_r) \), ... to \( v_{XY}(x) - v_{XY}(0°C) \) ... is called an electronic ice bath circuit.

These can be built from passive components or active devices.

An example of an ice-bath circuit ... will follow integrated temperature sensors.

Integrated Temperatures Sensors

Symbols: \( + \) (current source type) \( + \) (volt. source type).

Temperature range: about \(-100°C \) to \(200°C \). (Relatively narrow.)

Construction:

Transducer (usually diode) mounted ... in same package as conditioning circuit.

Principle of Operation

Temperature is sensed by some transducer.

Conditioning circuit converts temperature to ... a voltage or current ... (depending on type).

Voltage or current output is in user (engineer)-friendly form.

Desirable Characteristic

- Linear, human-oriented output.
  \( (E.g., \text{ current in microamps is temperature in Kelvins.}) \)

Undesirable Characteristics

- Narrow temperature range.
- Slow response to temperature changes.
- Fragile.

Typical Functions

Voltage type: \( H_1(x) = \frac{x \times 10 \text{ mV}}{K} \)

Current type: \( H_1(x) = \frac{x \mu A}{K} \)

Use

Current type must have at least several volts bias.

Current type best ... when resistance of leads may be significant, ... as when long leads are used.
Example: Thermocouple and Integrated Temperature Sensor

Design a circuit to convert a temperature, \( x \in [100 \, ^\circ C, 1760 \, ^\circ C] \), to a floating-point number stored in variable \( \text{tee} \), where \( \text{tee} = H(x) = x \cdot \frac{1}{100} \). Use the following:

- A Type-R thermocouple.
- A 2100-entry Type-R thermocouple table . . . stored as an array in the computer’s memory.
- An isothermal block . . . with an integrated temperature sensor . . . having response \( H_{TB}(x) = \frac{x}{AK} \).

The isothermal block will be exposed . . . to temperatures in the range \([5 \, ^\circ C, 50 \, ^\circ C]\).

Solution Plan:

- Design circuit.
- Choose component values.
- Write code to compute answer.

Instrumentation Amplifier Gain

ADC input cannot exceed 10 V.

Note: maximum occurs when \( x \) is at its maximum and \( T_i \) is at its minimum.

Value for \( H_{TB}(x) \) is found in Type-R thermocouple table.

- \( 0 \, V \leq H_{TB}(x) - H_{TB}(T_i) \leq 9 \, V \)
- \( 0 \, V \leq A[H_{TB}(x) - H_{TB}(T_i)] \leq 10 \, V \)

Minimum voltage: \( H_{TB}(100 \, ^\circ C) - H_{TB}(50 \, ^\circ C) = 0.351 \, mV \).

Maximum voltage: \( H_{TB}(1760 \, ^\circ C) - H_{TB}(5 \, ^\circ C) = 20.979 \, mV \).

Therefore, \( A < 477 \) must be satisfied.

Choose \( A = 450 \)

Makes use of more than 92% of ADC’s dynamic range. (Good.)

Resistance Of \( R_A \)

Constraint: \( 0 \, V \leq H_{TB}(T_i) \leq 9 \, V \)

Current-to-voltage circuit: \( H_{TB}(y) = yR_A \).

- \( 0 \, V \leq \frac{y}{A} R_A \leq 10 \, V \).
- \( R_A < \frac{10 \, V}{30.945 \, k\Omega} = 323.15 \, k\Omega \).

Choose: \( R_A = 20 \, k\Omega \).

Makes use of < 10% of ADC’s dynamic range. (Wasteful.)

Possible test or homework question: . . .

. . . “How can the circuit be modified . . . to make greater use of the ADC’s dynamic range?”
Interface Routine

Call the value read from the thermocouple input \( r_1 \) . . .
and call value from integrated temperature sensor \( r_2 \).

\[ r_1 = H_{ADC}(10\ V; 16)(H_{c1}(H_{TypeR}(x) - H_{TypeR}(T_r))) \]

Need to satisfy:

\[ H(H_{ADC}(10\ V; 16)(H_{c1}(H_{TypeR}(x) - H_{TypeR}(T_r)))) = H(x) = \frac{x}{T_r} \]

Let \( z = H(H_{ADC}(10\ V; 16)(H_{c1}(H_{TypeR}(x) - H_{TypeR}(T_r)))) \) and solve for \( x \).

\[ x = H_{TypeR}(\frac{1}{T_r} z + H_{TypeR}(T_r)) \]

\[ H_i(z) = H(x) = \frac{x}{T_r} - 273.15 \]

Next find \( T_r \).

\[ r_2 = H_{ADC}(H(10\ V; 16)) \]. Solving, \( T_r = \frac{r_2 V_{ADC}}{(2^8 - 1)A_{RA}} \)

Let function \( h_{TyR}(T) \) return the thermocouple voltage . . .
. . . at temperature \( T \) with reference temperature 0°C.

Let function \( h_{TyR}(v) \) return the thermocouple temperature . . .
. . . when the measured voltage is \( v \) with reference temperature 0°C.

Then:

\[
\begin{align*}
\text{double } & t_{\text{ref}} = r_2 \times 7.6293\times10^{-9}; /* = r_2 \times \frac{V_{ADC}}{(2^8 - 1)A_{RA}} */ \\
\text{double } & t_{ee} = h_{TyR}(r_1 \times 3.390\times10^{-7} + h_{TyR}(t_{\text{ref}})) - 273.15;
\end{align*}
\]