Raison d’être: convert tiny changes in resistance to voltage.

Shown with an instrumentation amplifier.
Like an ideal op-amp but with finite gain.

Gain of instrumentation amplifier denoted by $A$.

$$v_o = A(v_+ - v_i).$$

The Wheatstone bridge consists of four arms.

$$v_o = A \left( \frac{R_B}{R_A + R_B} - \frac{R_D}{R_C + R_D} \right) v_E.$$
Transducer can be placed in one, two, or four arms.

Typical function: \( H_t(x) = R(1 + xk) \), \( xk \ll 1 \)
where \( R \) is the nominal resistance of the transducer and \( k \) is a constant.

For simplicity write function as: \( H_t(x) = R + R_s \),
where \( R \) is independent of the process-variable value and \( R_s \) is dependent on the process-variable value.

Typically, \( R \gg R_s \).

Usually, need to convert \( R_s \) to a voltage.
Complementary Pairs

Frequently, transducer pairs can have *complementary responses*. If so, there are two (usually identical) transducers…

…positioned so they react *oppositely* to the process variable…

…so that when their responses are *subtracted*…

…their response to the process variable *add*…

…and unwanted quantities *cancel out*.

For example, consider:

\[ H_{t1}(x) = R(1 + xk) \text{ and } H_{t2}(x) = R(1 - xk). \]

Sum: \[ H_{t1}(x) + H_{t2}(x) = R. \] (Not helpful.)

Difference: \[ H_{t1}(x) - H_{t2}(x) = 2xk. \] (Much better.)
One-Transducer Configuration

Arm B: \( H_t(x) = R + R_s = R(1 + xk) \).

Other Arms: Resistor of value \( R \).

\[
v_o = A \left( \frac{R_s}{2(2R + R_s)} \right) v_E \approx A \frac{R_s}{4R} v_E = A \frac{xk}{4} v_E.
\]
Two-Transducer Configuration

Arm A: \( H_{t2}(x) = R - R_s = R(1 - xk) \).

Arm B: \( H_{t1}(x) = R + R_s = R(1 + xk) \).

Other Arms: Resistor of value \( R \).

\[
v_o = A \frac{R_s}{2R} v_E = A \frac{xk}{2} v_E.
\]

As one might expect, twice as sensitive.
Four-Transducer Configuration

Arms A and D:  \[ H_{t2}(x) = R - R_s = R(1 - xk). \]

Arms B and C:  \[ H_{t1}(x) = R + R_s = R(1 + xk). \]

\[ v_o = A \frac{R_s}{R} v_E = A x k v_E. \]
Wheatstone Bridge Transfer Functions

Goal

Let $R_t = R \pm R_s = R(1 \pm xk)$ be the transducer response(s).

Assume bridge designed properly.

Need to find two functions:

$$H_c(R_t) = \ldots \quad \text{and} \quad H_c(R_s) = \ldots$$

Both functions are equivalent.

Choose whichever is more convenient.

Four-Transducer Configuration

$$H_c(R_s) = v_o = A \frac{R_s}{R} v_E.$$ 

Let $R_t = R + R_s$. Then $R_s = R_t - R$.

$$H_c(R_t) = A \left( \frac{R_t}{R} - 1 \right) v_E.$$ 

Two-Transducer Configuration

$$H_c(R_s) = v_o = \frac{A}{2} \frac{R_s}{R} v_E.$$ 

$$H_c(R_t) = A \left( \frac{R_t}{R} - 1 \right) v_E.$$ 

One-Transducer Configuration

$$H_c(R_s) = v_o = \frac{A}{4} \frac{R_s}{R} v_E.$$ 

$$H_c(R_t) = A \left( \frac{R_t}{R} - 1 \right) v_E.$$
Wheatstone Bridge Sample Problem

Design a system with output \( v_o = H(x) \), where process variable \( x \) is strain and, \( x \in [0, 10^{-5}] \), and \( H(x) = 10^6 x V \).

Strain will be covered in more detail later.

For now, all we need to know is that strain is dimensionless.

Strain is measured by a strain gauge.

Strain gauges frequently used in complementary pairs.

Use strain gauges with response:

\[
H_t(\epsilon) = R(1 + \epsilon G_f),
\]

where \( \epsilon \) denotes strain and constant \( G_f = 2 \).

\((G_f \text{ called gauge factor, a dimensionless quantity.})\)

Position the two strain gauges to obtain response:

\[
H_t(x) = R(1 + x G_f) \quad \text{and} \quad H_{t'}(x) = R(1 - x G_f).
\]
Derivation of Conditioning Circuit Needed

A Wheatstone bridge is the obvious choice because transducer response is in form $R \pm R_s$.

Nevertheless, conditioning-circuit derivation will be presented.

$H(x) = H_c(H_t(x))$

(Analysis performed as though there were one transducer.)

$y = H_t(x) = R_t = R(1 + xG_f)$. Then $x = \frac{y}{R} - \frac{1}{G_f}$.

Then $H_c(y) = H\left(\frac{y}{R} - \frac{1}{G_f}\right) = 10^6 \frac{y}{R} - 1 \, \text{V}$.

Response for two-transducer configuration: $H_c(R_t) = \frac{A}{2} \left(\frac{R_t}{R} - 1\right)v_E$.

Choose $A$ and $v_E$ so that $\frac{A}{2}v_E = \frac{10^6}{G_f} \, \text{V}$ is satisfied.

For example, $v_E = 10 \, \text{V}$ and $A = 10^5$. 