The Wheatstone Bridge

\[ v_o = A \left( \frac{R_B}{R_A + R_B} - \frac{R_D}{R_C + R_D} \right) v_E. \]

**Raison d’être:** convert tiny changes in resistance to voltage.

Shown with an *instrumentation amplifier*.

Like an ideal op-amp but with finite gain.

Gain of instrumentation amplifier denoted by \( A \).

\[ v_o = A(v_+ - v_1). \]

The Wheatstone bridge consists of four arms.

Transducer can be placed in one, two, or four arms.

Typical function: \( H_t(x) = R(1 + xk) \).

where \( R \) is the nominal resistance of the transducer and \( k \) is a constant.

For simplicity write function as: \( H_t(x) = R + R_s \).

where \( R \) is independent of the process-variable value and \( R_s \) is dependent on the process-variable value.

Typically, \( R \gg R_s \).

Usually, need to convert \( R_s \) to a voltage.

**Complementary Pairs**

Frequently, transducer pairs can have complementary responses.

If so, there are two (usually identical) transducers... 

...positioned so they react oppositely to the process variable... 

...so that when their responses are subtracted... 

...their response to the process variable add... 

...and unwanted quantities cancel out.

For example, consider:

\[ H_{t1}(x) = R(1 + xk) \text{ and } H_{t2}(x) = R(1 - xk). \]

Sum: \( H_{t1}(x) + H_{t2}(x) = R \). (Not helpful.)

Difference: \( H_{t1}(x) - H_{t2}(x) = 2xk \). (Much better.)

**One-Transducer Configuration**

Arm B: \( H_t(x) = R + R_a = R(1 + xk) \).

Other Arms: Resistor of value \( R \).

\[ v_o = A \left( \frac{R_a}{2(2R + R_a)} \right) v_E \approx A \frac{R_a}{4R} v_E = A \frac{xk}{4} v_E. \]
Two-Transducer Configuration

Arm A: \( H_A(x) = R - R_s = R(1 - xk) \).

Arm B: \( H_B(x) = R + R_s = R(1 + xk) \).

Other Arms: Resistor of value \( R \).

\[ v_o = A \frac{R_s}{2R} v_E = A \frac{xk}{2} v_E. \]

As one might expect, twice as sensitive.

Four-Transducer Configuration

Arms A and D: \( H_A(x) = R - R_s = R(1 - xk) \).

Arms B and C: \( H_B(x) = R + R_s = R(1 + xk) \).

\[ v_o = A \frac{R_s}{R} v_E = Axk v_E. \]

Wheatstone Bridge Transfer Functions

Goal

Let \( R_t = R \pm R_s = R(1 \pm xk) \) be the transducer response(s).

Assume bridge designed properly.

Need to find two functions:

\[ H_c(R_t) = \ldots \] and \[ H_c(R_s) = \ldots \]

Both functions are equivalent. Choose whichever is more convenient.

Four-Transducer Configuration

\[ H_c(R_s) = v_o = A \frac{R_s}{2R} v_E. \]

Let \( R_t = R + R_s \). Then \( R_s = R - R_t \).

\[ H_c(R_t) = \frac{A}{2} \frac{R_s}{R} v_E = A \left( \frac{R_t}{R} - 1 \right) v_E. \]

Two-Transducer Configuration

\[ H_c(R_s) = v_o = A \frac{R_s}{2R} v_E. \]

\[ H_c(R_t) = \frac{A}{2} \frac{R_s}{R} v_E = A \left( \frac{R_t}{R} - 1 \right) v_E. \]

One-Transducer Configuration

\[ H_c(R_s) = v_o = A \frac{R_s}{4R} v_E. \]

\[ H_c(R_t) = \frac{A}{4} \frac{R_s}{R} v_E = A \left( \frac{R_t}{R} - 1 \right) v_E. \]

Wheatstone Bridge Sample Problem

Design a system with output \( v_o = H(x) \), where process variable \( x \) is strain and \( x \in [0, 10^{-5}] \), and \( H(x) = 10^6 x V \).

Strain will be covered in more detail later.

For now, all we need to know is that strain is dimensionless.

Strain is measured by a strain gauge.

Strain gauges frequently used in complementary pairs.

Use strain gauges with response:

\[ H_t(G_f) = R + R_s = R(1 + xG_f), \]

where \( \epsilon \) denotes strain and constant \( G_f = 2 \).

\( (G_f \) called gauge factor, a dimensionless quantity.\)

Position the two strain gauges to obtain response:

\[ H_t(x) = R(1 + xG_f) \] and \[ H_t(x) = R(1 - xG_f). \]
Derivation of Conditioning Circuit Needed

A Wheatstone bridge is the obvious choice because transducer response is in form $R \equiv R_c$.

Nevertheless, conditioning-circuit derivation will be presented.

$H(x) = H_c(H_t(x))$

(Analysis performed as though there were one transducer.)

$y = H_t(x) = R_t = R(1 + xG_f)$. Then $x = \frac{y}{G_f} - \frac{1}{G_f}$.

Then $H_c(y) = H\left(\frac{y}{G_f} - \frac{1}{G_f}\right) = 10^6 \frac{y}{G_f} - \frac{1}{G_f} V$.

Response for two-transducer configuration: $H_c(R_t) = \frac{A}{2} \left(\frac{R_t}{R_t} - 1\right)v_E$. Choose $A$ and $v_E$ so that $\frac{A}{2}v_E = 10^6 \frac{1}{G_f} V$ is satisfied.

For example, $v_E = 10 V$ and $A = 10^5$. 