1g

$A = (a_4 a_3 a_2 a_1 a_0)_2$

$B = (b_4 b_3 b_2 b_1 b_0)_2$

A, B unsigned

$$
\begin{array}{rccccccccl}
 & & & & & a_4 & a_3 & a_2 & a_1 & a_0 = A \\
 & & & & \times) & b_4 & b_3 & b_2 & b_1 & b_0 = B \\
\hline
 & & & & & a_4b_0 & a_3b_0 & a_2b_0 & a_1b_0 & a_0b_0 \\
 & & & & a_4b_1 & a_3b_1 & a_2b_1 & a_1b_1 & a_0b_1 & \\
 & & & a_4b_2 & a_3b_2 & a_2b_2 & a_1b_2 & a_0b_2 & & \\
 & & a_4b_3 & a_3b_3 & a_2b_3 & a_1b_3 & a_0b_3 & & & \\
 & a_4b_4 & a_3b_4 & a_2b_4 & a_1b_4 & a_0b_4 & & & & \\
\hline
\end{array}
$$

Matrix of summands

Each term $a_i b_j$ is called summand.

2 g

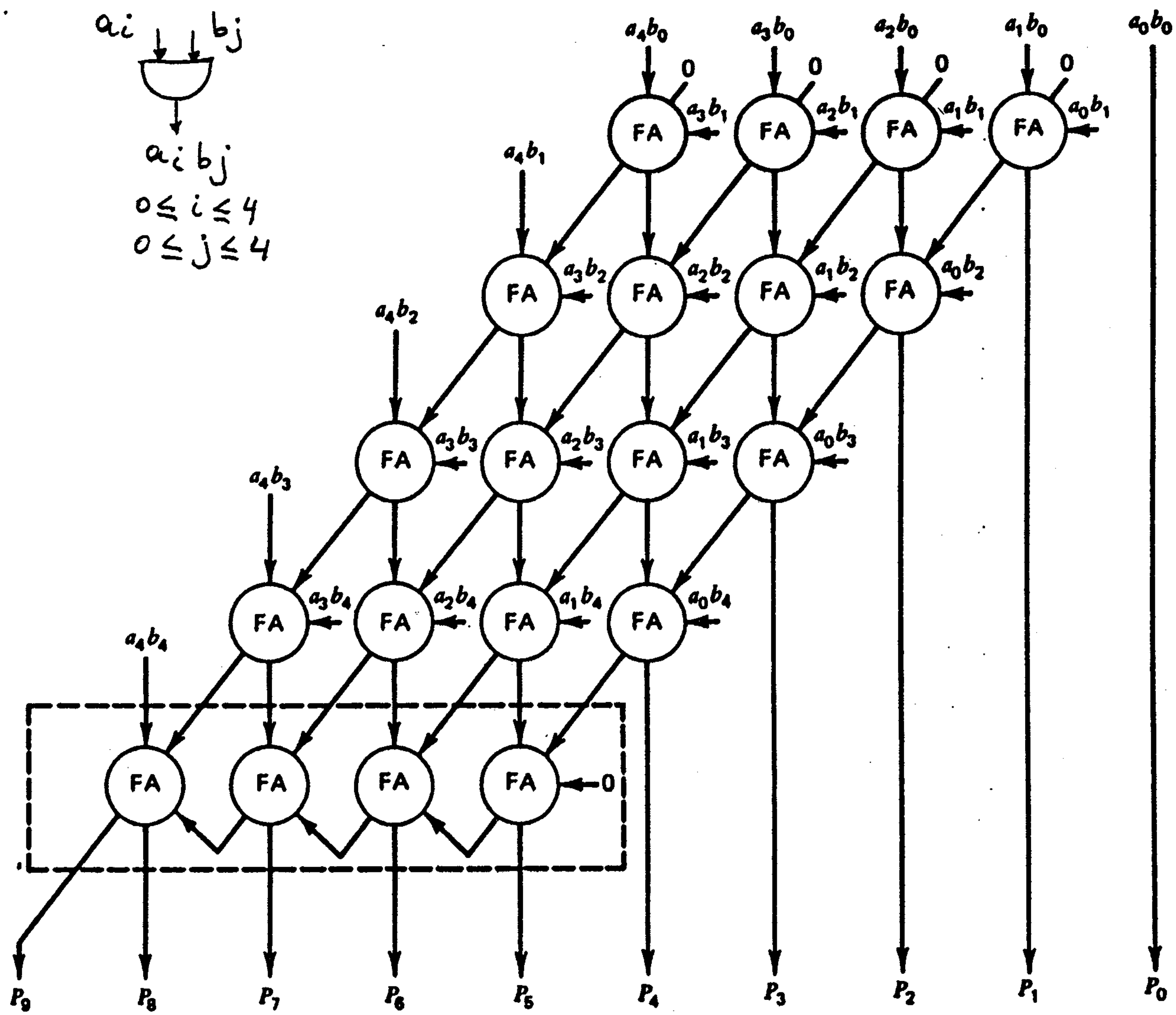
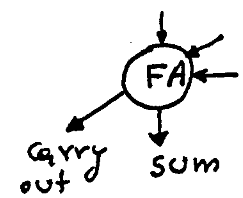
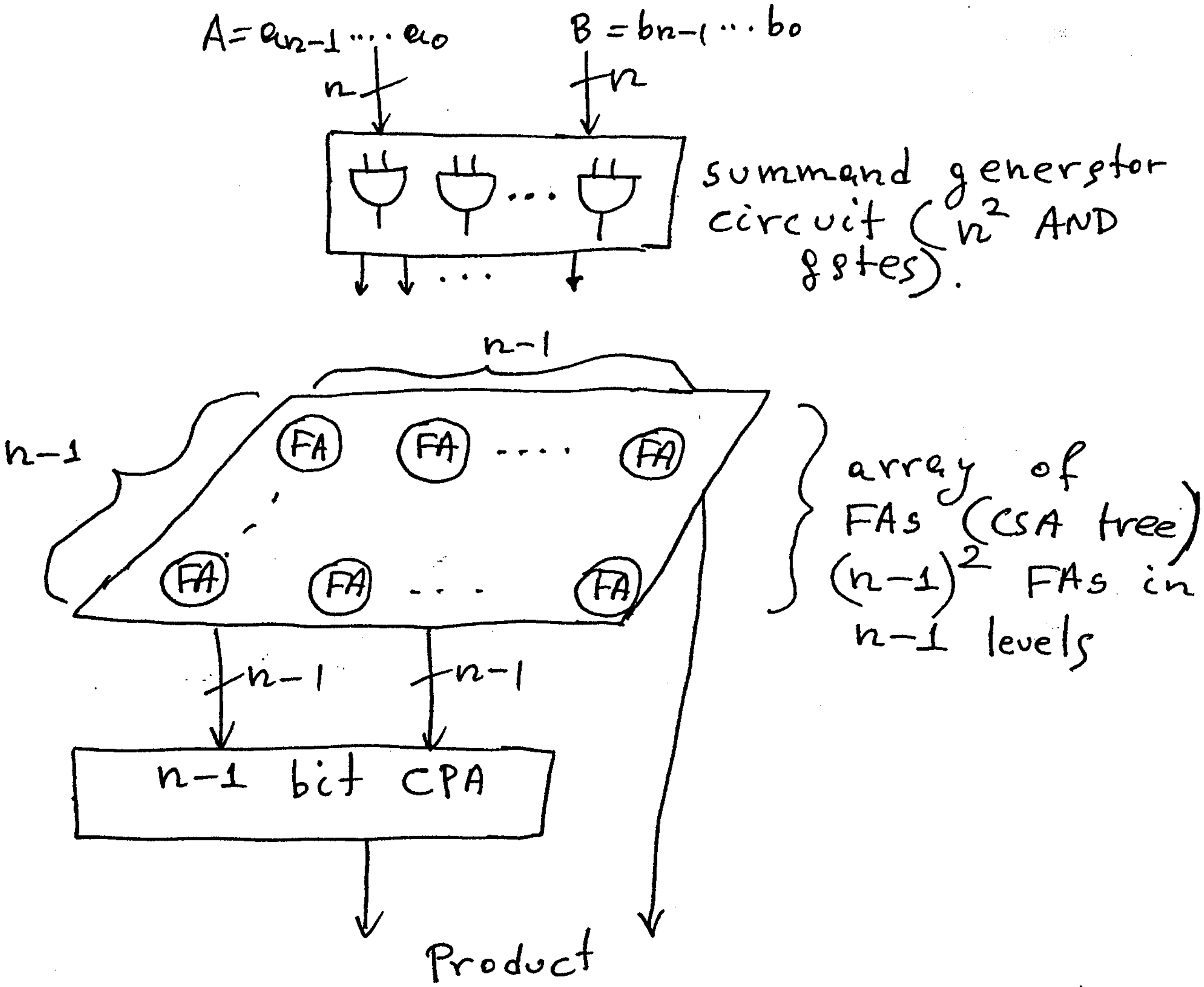


Figure 6.3 The schematic circuit diagram of a 5-by-5 unsigned array multiplier (Braun [5]).

Note



n-by-n Unsigned Cellular Array Multiplier



Cost : n^2 AND + $(n-1)^2$ FA + CPA

Delay : $D_{AND} + (n-1) D_{FA} + D_{CPA}$; where D_{AND}, D_{FA} and D_{CPA} are delays of AND gate, FA and (n-1) bit CPA respectively.

1 h

$A = (a_3)\ a_2\ a_1\ a_0$

$B = (b_3)\ b_2\ b_1\ b_0$

A, B are 4-bit signed numbers (2's complement system used)

* (a_3) and (b_3) indicate that bits a_3 and b_3 have negative weights of -2^3

$$
\begin{array}{ccccccc}
 & & & (a_3) & a_2 & a_1 & a_0 = A \\
 & & \times) & (b_3) & b_2 & b_1 & b_0 = B \\
\hline
 & & & (a_3 b_0) & a_2 b_0 & a_1 b_0 & a_0 b_0 \\
 & & (a_3 b_1) & a_2 b_1 & a_1 b_1 & a_0 b_1 & \\
 & (a_3 b_2) & a_2 b_2 & a_1 b_2 & a_0 b_2 & & \\
a_3 b_3 & (a_2 b_3) & (a_1 b_3) & (a_0 b_3) & & & \\
\hline
\end{array}
$$

Matrix of summands

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Derivations for the Baugh - Wooley multiplier

Lemma 1: Let $A=(a_{n-1}a_{n-2}\cdots a_1a_0)_2$ be a signed binary number (the 2's complement system is used to represent signed numbers). Then the value of the number A is $A_v = -a_{n-1}\cdot 2^{n-1} + \sum_{i=0}^{n-2} a_i 2^i$. ($a_{n-1}$ = sign bit).

Proof:

Case (i): $A \geq 0$

In this case $a_{n-1}=0$ and $A_v = 0 + \sum_{i=0}^{n-2} a_i 2^i$ (which is expected).

Case (ii): $A<0$

Here $a_{n-1}=1$. The value of A is

$$A_v = -(2^n-(a_{n-1}a_{n-2}\cdots a_1a_0)_2) = -(2^n - a_{n-1}2^{n-1} - \sum_{i=0}^{n-2} a_i 2^i)$$
$$= -(2^n - 2^{n-1} - \sum_{i=0}^{n-2} a_i 2^i) = -(2^{n-1} - \sum_{i=0}^{n-2} a_i 2^i)$$
$$= -2^{n-1} + \sum_{i=0}^{n-2} a_i 2^i = \boxed{-a_{n-1}\cdot 2^{n-1} + \sum_{i=0}^{n-2} a_i 2^i}$$

Lemma 2: Let $A=(a_{n-1}\cdots a_1a_0)_2$ be a signed binary number with a_{n-1} being its sign bit (2's compl. system used to represent signed numbers). Then the value of $-A$ is

$$-A_v = -\overline{a_{n-1}}\cdot 2^{n-1} + \left[\sum_{i=0}^{n-2} \overline{a_i}\cdot 2^i\right] + 1.$$

Proof:

Lemma 1 indicates that $A_v = -a_{n-1}\cdot 2^{n-1} + \sum_{i=0}^{n-2} a_i\cdot 2^i$

Multiplying both sides of the above with -1 we get

$$-A_v = a_{n-1}\cdot 2^{n-1} - \sum_{i=0}^{n-2} a_i\cdot 2^i = a_{n-1}\cdot 2^{n-1} + (2^{n-1} - 2^{n-1}) - \sum_{i=0}^{n-2} a_i\cdot 2^i$$

2 i

$$= (a_{n-1} - 1) \cdot 2^{n-1} + 2^{n-1} - \sum_{i=0}^{n-2} a_i \cdot 2^i$$

$$= -(1 - a_{n-1}) \cdot 2^{n-1} + \Big[\underbrace{(2^{n-2} + 2^{n-3} + \cdots + 2^1 + 2^0) + 1}_{2^{n-1}}\Big] - \sum_{i=0}^{n-2} a_i\, 2^i$$

$$= -(1 - a_{n-1})\, 2^{n-1} + 1 + \sum_{i=0}^{n-2} 2^i - \sum_{i=0}^{n-2} a_i \cdot 2^i$$

$$= -(1 - a_{n-1}) \cdot 2^{n-1} + \Big[\sum_{i=0}^{n-2} (1 - a_i)\, 2^i\Big] + 1 = \boxed{-\overline{a_{n-1}} \cdot 2^{n-1} + \Big[\sum_{i=0}^{n-2} \overline{a_i} \cdot 2^i\Big] + 1}$$

(Observe that $\overline{a_i} = 1 - a_i$).

After the two lemmas are stated and proven, the Baugh-Wooley multiplier can easily be derived.

<u>Baugh - Wooley derivations</u> :

Let $A = (a_{m-1}\, a_{m-2} \cdots a_1\, a_0)_2$ and $B = (b_{n-1}\, b_{n-2} \cdots b_1\, b_0)_2$ be two signed binary numbers with a_{m-1} and b_{n-1} being their sign bits. Then, according to Lemma 1, their values are

$$A_v = -a_{m-1} \cdot 2^{m-1} + \sum_{i=0}^{m-2} a_i \cdot 2^i$$

$$B_v = -b_{n-1} \cdot 2^{n-1} + \sum_{j=0}^{n-2} b_j \cdot 2^j .$$

The value of their product would then be

$$P_v = A_v \times B_v = \Big(-a_{m-1} \cdot 2^{m-1} + \sum_{i=0}^{m-2} a_i \cdot 2^i\Big) \times \Big(-b_{n-1} \cdot 2^{n-1} + \sum_{j=0}^{n-2} b_j \cdot 2^j\Big)$$

$$= a_{m-1} \cdot b_{n-1} \cdot 2^{m+n-2} + \sum_{i=0}^{m-2} \sum_{j=0}^{n-2} a_i \cdot b_j \cdot 2^{i+j}$$

$$- a_{m-1} \cdot 2^{m-1} \Big(\sum_{j=0}^{n-2} b_j \cdot 2^j\Big) - b_{n-1} \cdot 2^{n-1} \cdot \Big(\sum_{i=0}^{m-2} a_i \cdot 2^i\Big) \qquad (1)$$

In the above eq. (1) the term $a_{m-1} \cdot 2^{m-1} \cdot \sum_{j=0}^{n-2} b_j\, 2^j$

can be rewritten as

$$2^{m-1} \cdot \sum_{j=0}^{n-2} a_{m-1} \cdot b_j \cdot 2^j = 2^{m-1} \cdot \left(-0 \times 2^n + 0 \times 2^{n-1} + \sum_{j=0}^{n-2} a_{m-1} \cdot b_j \cdot 2^j\right) \quad (2)$$

Our goal is to try to compute $-a_{m-1} \cdot 2^{m-1} \cdot \sum_{j=0}^{n-2} b_j \cdot 2^j$ which is needed by equation (1). From eq. (2) it is clear that if $a_{m-1} = 0$ then $a_{m-1} \cdot 2^{m-1} \cdot \sum_{j=0}^{n-2} b_j \cdot 2^j = 0$. Thus

$$\boxed{-a_{m-1} \cdot 2^{m-1} \cdot \sum_{j=0}^{n-2} b_j \cdot 2^j = 0 \quad \text{if } a_{m-1} = 0} \quad (3)$$

On the other hand if $a_{m-1} = 1$, then Lemma 2 applied on eq. (2) will give

$$-a_{m-1} \cdot 2^{m-1} \cdot \sum_{j=0}^{n-2} b_j \cdot 2^j = 2^{m-1} \cdot \left(-2^n + 2^{n-1} + 1 + \sum_{j=0}^{n-2} \overline{a_{m-1} \cdot b_j} \cdot 2^j\right)$$

$$= 2^{m-1} \cdot \left(-2^n + 2^{n-1} + 1 + \sum_{j=0}^{n-2} \overline{b_j} \cdot 2^j\right) \quad \text{or}$$

$$\boxed{-a_{m-1} \cdot 2^{m-1} \cdot \sum_{j=0}^{n-2} b_j \cdot 2^j = 2^{m-1} \cdot \left(-2^n + 2^{n-1} + 1 + \sum_{j=0}^{n-2} \overline{b_j} \cdot 2^j\right) \quad \text{if } a_{m-1} = 1} \quad (4)$$

Combining eqs (3) and (4) into one equation one gets

$$\boxed{-a_{m-1} \cdot 2^{m-1} \cdot \sum_{j=0}^{n-2} b_j \cdot 2^j = 2^{m-1} \cdot \left(-2^n + 2^{n-1} + \overline{a_{m-1}} \cdot 2^{n-1} + a_{m-1} + \sum_{j=0}^{n-2} a_{m-1} \cdot \overline{b_j} \cdot 2^j\right)} \quad (5)$$

To show the validity of eq (5) observe that

for $a_{m-1} = 0$, the right hand side of (5) gives $(-2^n + 2^{n-1} + 2^{n-1} + 0 + 0) = 0$ (which is in agreement with eq (3))

while if $a_{m-1} = 1$, the right hand side of eq (5) becomes

$$2^{m-1}\cdot\left(-2^{n}+2^{n-1}+1+\sum_{j=0}^{n-2}\overline{b_j}\cdot 2^{j}\right)$$, which is in agreement with eq. (4).

In a similar way we can derive

$$\boxed{-b_{n-1}\cdot 2^{n-1}\cdot\sum_{i=0}^{m-2}a_i\cdot 2^{i} = 2^{n-1}\cdot\left(-2^{m}+2^{m-1}+\overline{b_{n-1}}\cdot 2^{m-1}+b_{n-1}+\sum_{i=0}^{m-2}b_{n-1}\cdot\overline{a_i}\cdot 2^{i}\right)} \quad (6)$$

Based on eqs. (5) and (6), equation (1) can be rewritten as

$$\boxed{\begin{aligned}P_V = A_V\times B_V &= a_{m-1}\cdot b_{n-1}\cdot 2^{m+n-2}+\sum_{i=0}^{m-2}\sum_{j=0}^{n-2}a_i\cdot b_j\cdot 2^{i+j}\\ &+2^{m-1}\cdot\left(-2^{n}+2^{n-1}+\overline{a_{m-1}}\cdot 2^{n-1}+a_{m-1}+\sum_{j=0}^{n-2}a_{m-1}\cdot\overline{b_j}\cdot 2^{j}\right)\\ &+2^{n-1}\cdot\left(-2^{m}+2^{m-1}+\overline{b_{n-1}}\cdot 2^{m-1}+b_{n-1}+\sum_{i=0}^{m-2}b_{n-1}\cdot\overline{a_i}\cdot 2^{i}\right)\end{aligned}}$$