EE 3755

Computer Arithmetic

Handout # 7

Division of fractions

Problem: Consider two n-bit (binary) fractions $f_1 = .a_{n-1} \cdots a_1 a_0$ and $f_2 = .b_{n-1} \cdots b_1 b_0$. Explain how you can use the procedure for dividing a $2n$-bit integer dividend by an n-bit integer divisor in order to compute $f_3 = \frac{f_1}{f_2}$, where f_3 must ~~be~~ also be an n-bit fraction; (f_3 must be the best n-bit approximation of $\frac{f_1}{f_2}$).

Answer: Consider the integers A; B where

integer $A = a_{n-1} \cdots a_1 a_0 \overbrace{00 \cdots 00}^{n \text{ zeros}}$.

integer $B = b_{n-1} \cdots b_1 b_0$.

Divide A by B to get an n-bit quotient Q and an n-bit remainder R.

Then $A = B \times Q + R$ or

$$\frac{A}{B} = Q + \frac{R}{B}.$$

Recalling that $R < B$, (or $0 \leq \frac{R}{B} < 1$), it is obvious that the best integer approximation of $\frac{A}{B}$ is Q' or

$$\frac{A}{B} \cong Q' \quad \text{where} \quad Q' = \begin{cases} Q & \text{if } \frac{R}{B} < \frac{1}{2} \\ Q+1 & \text{if } \frac{R}{B} \geq \frac{1}{2} \end{cases}$$

Obviously $A \cong B \times Q'$ (1).

Let Q' be $Q' = q'_{n-1} \cdots q'_1 q'_0 .$

Then $\boxed{f_3 = \frac{f_1}{f_2} \cong .q'_{n-1} \cdots q'_1 q'_0}$

<u>Question</u>: why does the above work?

<u>Answer</u>: Eq. (1) dictates

$$(a_{n-1} \cdots a_1 a_0 00 \cdots 00.)_2 \cong (b_{n-1} \cdots b_1 b_0.)_2 \times (q'_{n-1} \cdots q'_1 q'_0 .)_2$$

or

$$\frac{(a_{n-1}\cdots a_0 0\cdots 0.)_2}{2^{2n}} \cong \frac{(b_{n-1}\cdots b_0.)_2}{2^n} \times \frac{(q'_{n-1}\cdots q'_0.)_2}{2^n}$$

or

$$(.a_{n-1}\cdots a_0 0\cdots 0)_2 \cong (.b_{n-1}\cdots b_0)_2 \times (.q'_{n-1}\cdots q'_0)_2$$

or $f_1 \cong f_2 \times (.q'_{n-1}\cdots q'_0)_2$

or $\boxed{\dfrac{f_1}{f_2} \cong .q'_{n-1}\cdots q'_1 q'_0}$

<u>Example</u>: Compute $f_3 = \dfrac{f_1}{f_2}$ where f_1, f_2 are the following 4-bit fractions: $f_1 = .1000$; $f_2 = .1100$

<u>Solution</u>: Here $A = (10000000.)_2 = 128$; $B = (1100.)_2 = 12$. Dividing A by B

we get quotient $Q = 10 =$
$= (1010)_2$ and remainder $R = 8 =$
$(1000)_2$. Here $\frac{R}{B} = \frac{8}{12} > \frac{1}{2}$.
Therefore $Q' = Q + 1 = 11 = (1011.)_2$
and $\boxed{f_3 = \frac{f_1}{f_2} \cong (.1011)_2}$

To double check see that $f_1 = .1000 =$
$\frac{1}{2}$; $f_2 = .1100 = \frac{1}{2} + \frac{1}{4} = 3/4$. The
actual f_1/f_2 is $\frac{f_1}{f_2} = \frac{1/2}{3/4} = \frac{4}{6} = .666\cdots6$

What we got as an approximation was
$.1011 = \frac{1}{2} + \frac{1}{8} + \frac{1}{16} = \frac{11}{16} = .6875$
The obtained result $(.1011)_2 = .6875$
is the closest 4-bit fractional approximation of $f_1/f_2 = .666\cdots66$.
Observe that $(.1010)_2 = \frac{1}{2} + \frac{1}{8} = .625$
and $.6875$ is closer to $.666\cdots66$
than $.625$ is.