

EE 3755, Fall 2011

Homework #1 solutions

~~HW#1 solution~~
[1] Here the numbers X and Y are of different signs. Therefore, we have to perform the subtraction

$$(\text{magnitude of } X) - (\text{magnitude of } Y) =$$

$$(1011011)_2 - (1101001)_2 =$$

$$= (1011011) + 2's \text{ compl. of } (1101001)$$

$$= 1011011$$

$$+ 0010111$$

$$\hline 01110010$$

$$\hookrightarrow c=0 \Rightarrow \text{result} < 0 \Rightarrow (\text{mag. of } X) - (\text{mag. of } Y) < 0$$

$$\Rightarrow \text{mag. of } X < \text{mag. of } Y$$

$$\text{So sign bit of result} = \text{sign bit of } Y = 1$$

$$\text{and magnitude of } X+Y =$$

$$= 2's \text{ compl. of } (1110010) = 0001110$$

$$\text{Thus, } X+Y = (10001110)_2 = (-14)_{10}$$

2 ~~HW #1 solution (3755)~~ ②

B A d
0000000 100101 0 multiplication

+ 1101110

010 $\Rightarrow$ add 1x mult/carry,
double right shift

1101110 100101 0

1111101 101001 0

010 $\Rightarrow$ add 1x mult/carry,
double right shift

+ 1101110

1110100 101001 0

1111101 011010 0

100 $\Rightarrow$ add -2x mult/carry
/ double right shift.

0100100

0011110 011010 0

000111 100110 1

$\Rightarrow$ product =

$$(000111100110)_2 = (486)_{10}$$

$$= (-18) \times (-27)$$

3

C

R

Q

X

01010 | 10001

multiplication

10101 | 0001

shift left/subtr. B

C

10011

1

01000 | 00011

shift left/subtr. B

C

10000 | 0011

1

10011

00011 | 0011

shift left/subtr. B

C

10011

0

11001 | 0110

shift left/add B

C
0

11010 | 1110

01101

11111 | 1100

shift left/add B

11111 | 1100

01101

1

01100 | 11001

* Restore is not needed

$$P = (01100)_2 = 12$$

$$Q = (11001)_2 = 25$$

4

C
X

R	Q
01010	00011

multiplier
shift left/subtr. B

C
1

10100	0011
-------	------

10011

00111	00111
-------	-------

shift left/subtr. B

01110	0111
-------	------

C
1

10011

00001	01111
-------	-------

shift left/subtr. B

C
0

00010	1111
-------	------

10011

10101	11110
-------	-------

shift left/add B

01011	1110
-------	------

C
0

01101

11000	11100
-------	-------

shift left/add B

10001	1100
-------	------

C
0

01101

11110	11000
-------	-------

restore; (i.e. add B)

01101

01011	11000
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$$R = (01011)_2 = 11$$

$$Q = (11000)_2 = 24$$

5 The range of the fraction ⑤
 is $0.5 \leq f \leq 1 - 2^{-48}$

The range of the exponent is

$$-2^{10} \leq e \leq 2^{10} - 1 \quad \text{or} \quad -1024 \leq e \leq 1023$$

So the positive dynamic range is

$$0.5 \times 2^{-1024} \leq A^+ \leq (1 - 2^{-48}) \times 2^{1023}$$

The negative dynamic range is

$$-(1 - 2^{-48}) \times 2^{1023} \leq A^- \leq -0.5 \times 2^{-1024}$$

6 1. Align/adjust

$$e_1 - e_2 = e_1 + 2^s \text{ complement of } e_2 =$$

$$= 0111$$

$$+) 0111$$

$$01110$$

$$\hookrightarrow c=0 \Rightarrow e_1 - e_2 < 0 \Rightarrow e_1 < e_2 \text{ and}$$

$$e_2 - e_1 = 2^s \text{ compl. of } (1110) = (0010)_2 = (2)_{10}$$

Thus A_1 becomes

$$A_1: \begin{array}{c|c|c} s_1 & e_2 & f_1' \\ \hline 0 & 1001 & 00111100 \end{array}$$

2. Subtract fractions

Since A_1 and A_2 are of different signs and $A_1 + A_2$ needs to be performed, a subtraction must take place.

$$f_1' - f_2 = f_1' + 2^s \text{ complement of } f_2 =$$

$$= \begin{array}{r} .00111100 \\ +) .01101110 \\ \hline 0.10101010 \end{array}$$

$$\hookrightarrow c=0 \Rightarrow f_1' - f_2 < 0 \Rightarrow f_1' < f_2$$

Since f_2 is the larger fraction, the result $A_3 = A_1 + A_2$ must have as a sign bit the sign bit of A_2 (negative sign). The fraction of $A_3 = A_1 + A_2$ will be the 2's compl. of $(10101010) = 01010110$.

Therefore

$$A_3: \boxed{1 \mid 1001 \mid 01010110}$$

3. Postnormalize

After postnormalization we get

$$A_3: \begin{array}{c|c|c} s_3 & e_3 & f_3 \\ \hline 1 & 1000 & 10101100 \end{array}$$

4. Check for exponent underflow

No exponent underflow occurred

See that $e_3 = (1000)_2 = 8 \in [0 \ 15]$