EE 3755

Fall 03

HW# 1 Solutions

[1] Here the two numbers X and Y are of different signs and X+Y needs to be performed. We thus perform the following subtraction:

(magnitude of X) − (magn. of Y) = (11100) − (11110)
= (11100) + 2's compl. of (11110) =

$$
\begin{array}{rl}
= & 11100 \\
+) & 00010 \\
\hline
 & 0\,11110
\end{array}
$$

$\hookrightarrow C = 0 \Rightarrow$ result $< 0 \Rightarrow$ (magn. of X) − (mag. of Y) < 0
$\Rightarrow$ magn. of X $<$ magn. of Y.

Therefore, sign bit of result = sign bit of Y = 1 and magnitude of (X+Y) = 2's compl. of (11110) = (00010). Thus $X+Y = (100010)_2 = (-2)_{10}$.

$\boxed{2}$ Here $n=6$, multiplier $= (-27)_{10} =$
$= (100101)_2$; multiplicand $= (-22)_{10} = (101010)_2$

Since three bits are to be examined at a time, the field B (left of multiplier field) must be of length $6+1 = 7$ bits

	B	A	d
Initialization	0000000	100101	0

+) 1101010

$\underbrace{010}$ → 010 → add 1× mult/cand and then do 2-bit right shift

B	A	d
1101010	100101	0

	B	A	d
result of 1st cycle	1111010	101001	0

+) 1101010

→ 010 → add 1× mult/cand and do 2-bit right sh

B	A	d
1100100	101001	0

	B	A	d
result of 2nd cycle	1111001	001010	0

+) 0101100

→ 100 → add $-2\times$ mult/cand and do 2-bit right shift

B	A	d
0100101	001010	0

B	A	d
0001001	010010	1

→ product =

$= (001001010010)_2 = 594 =$
$= (-22) \times (-27)$

$\boxed{3}$ Here the dividend is $A = (0000001101)_2 = (+13)_{10}$; the divisor is $B = (00101)_2 = (+5)_{10}$ and $n = 5$. Also, $-B$ = 2's complement of $B = (11011)_2$

C: $\boxed{X}$ | R: 00000 | Q: 01101 — Initialization

00000 | 1101 — shift R, Q left

+) 11011 — subtract B

C = $\boxed{0}$ 11011

11011 | 11010

(1st division cycle)

10111 | 1010 — shift R, Q left

+) 00101 — add B (since previous cycle gave c=0)

C = $\boxed{0}$ 11100

11100 | 10100

(2nd division cycle)

11001 | 0100 — shift R, Q left

+) 00101 — add B (since previous cycle gave c=0)

C = $\boxed{0}$ 11110

11110 | 01000

(3rd division cycle)

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4th division cycle

	R	Q	
	11100	1000	shift R, Q left
+)	00101		add B (since previous cycle gave c=0)
C = 1	00001		
	00001	10001	

5th division cycle

	R	Q	
	00011	0001	shift R, Q left
+)	11011		subtr. B (since previous cycle gave c=1)
C = 0	11110		
	11110	00010	

Restore cycle

	R	Q	
+)	00101		restore remainder by adding B to it since 5th division cycle gave c=0
	00011		
	00011	00010	

So remainder is $R = (00011)_2 = 3$ while quotient is $Q = (00010)_2 = 2$. Double check to see that $B \times Q + R = 5 \times 2 + 3 = 13 = A$

$\boxed{4}$ Here the dividend is $A = (0000010001)_2 = (+17)_{10}$; the divisor is $B = (00011)_2 = (+3)_{10}$ and $n = 5$. Also, $-B$ = 2's complement of $B = (11101)_2$.

C	R	Q	
$\boxed{X}$	00000	10001	Initialization

1st division cycle

	R	Q	
	00001	0001	shift R, Q left
+)	11101		subtract B
C = $\boxed{0}$	11110		
	11110	00010	

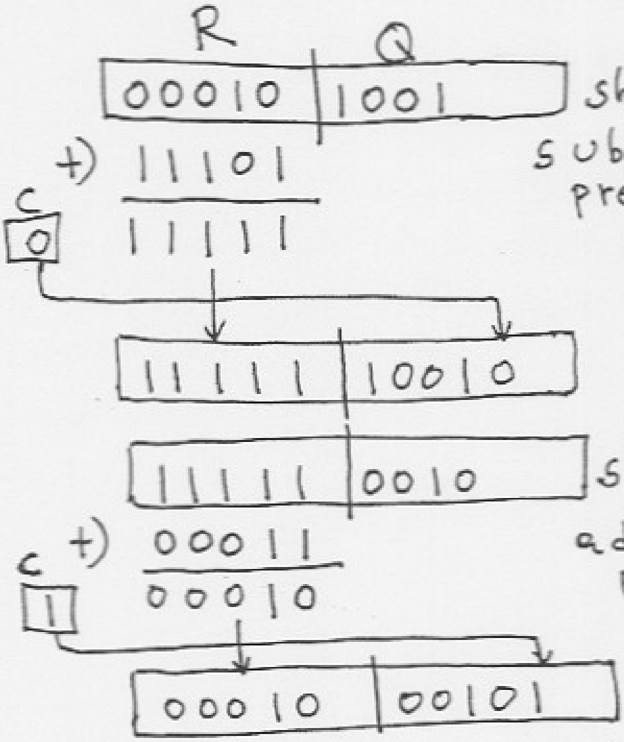
2nd division cycle

	R	Q	
	11100	0010	shift R, Q left
+)	00011		add B (since previous cycle gave c = 0)
C = $\boxed{0}$	11111		
	11111	00100	

3rd division cycle

	R	Q	
	11110	0100	shift R, Q left
+)	00011		add B (since previous cycle gave c = 0)
C = $\boxed{1}$	00001		
	00001	01001	

→ next page



shift R, Q left
subtr. B (since previous cycle gave c=1)

4th division cycle

shift R, Q left
add B (since previous cycle gave c=0)

5th division cycle

No restore cycle is necessary since the carry out of the 5th division cycle is 1. Here remainder is $R = (00010)_2 = 2$ while quotient is $Q = (00101)_2 = 5$. Double check to see that $B \times Q + R = 3 \times 5 + 2 = 17 = A$.

5 The range of the fraction is

$$\boxed{0.5 \le f \le 1 - 2^{-29}}$$

The range of the exponent is

$-2^9 \le e \le 2^9 - 1$ or $\boxed{-512 \le e \le 511}$

Thus the positive FLP dynamic range is

$$\boxed{0.5 \times 2^{-512} \le A^+ \le (1 - 2^{-29}) \times 2^{511}}$$

The negative FLP dynamic range is

$$\boxed{-(1 - 2^{-29}) \times 2^{511} \le A^- \le -0.5 \times 2^{-512}}$$

6 1. Align/adjust

$e_1 - e_2 = e_1 + 2$'s complement of $e_2 =$

$$\begin{array}{rl} = & 0111 \\ +) & 0111 \\ \hline & 0\,1110 \end{array}$$

$\hookrightarrow c = 0 \Rightarrow e_1 - e_2 < 0 \Rightarrow e_1 < e_2$ and $e_2 - e_1$

$= 2$'s complement of $(1110) = (0010)_2 = (2)_{10}$

Thus A_1 becomes

$$A_1: \begin{array}{c|c|c} s_1 & e_2 & f_1' \\ \hline 0 & 1001 & 00111100 \end{array}$$

2. Subtract fractions

Since A_1 and A_2 are of different signs and $A_1 + A_2$ needs to be performed, a subtraction must take place.

$f_1' - f_2 = f_1' + 2$'s complement of $f_2 =$

$$\begin{array}{r} .00111100 \\ +)\ .01101110 \\ \hline 0\ .10101010 \end{array}$$

$\hookrightarrow c = 0 \Rightarrow f_1' - f_2 < 0 \Rightarrow f_1' < f_2$. Since f_2 is the larger fraction, the result $A_3 = A_1 + A_2$ must have as a sign bit the sign bit of A_2 (negative sign). The fraction of $A_3 = A_1 + A_2$ will be the 2's complement of (10101010) = 01010110

Therefore

$$A_3: \boxed{1 \mid 1001 \mid 01010110}$$

3. Postnormalize

After postnormalization we get

A_3:

s_3	e_3	f_3
1	1000	10101100

4. Check for exponent underflow

No exponent underflow occured.

See that $e_3 = (1000)_2 = 8 \in [0 \quad 15]$

$\boxed{7}$

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(a)

$$G_5^* = G_{23} + G_{22} \cdot P_{23} + G_{21} \cdot P_{22} \cdot P_{23} + G_{20} \cdot P_{21} \cdot P_{22} \cdot P_{23}$$

(b) $P_6^* = P_{27} \cdot P_{26} \cdot P_{25} \cdot P_{24}$

(c)

$$c_{23} = G_5^* + G_4^* \cdot P_5^* + G_3^* \cdot P_4^* \cdot P_5^* + G_2^* \cdot P_3^* \cdot P_4^* \cdot P_5^*$$
$$+ G_1^* \cdot P_2^* \cdot P_3^* \cdot P_4^* \cdot P_5^* + G_0^* \cdot P_1^* \cdot P_2^* \cdot P_3^* \cdot P_4^* \cdot P_5^*$$
$$+ c_{-1} \cdot P_0^* \cdot P_1^* \cdot P_2^* \cdot P_3^* \cdot P_4^* \cdot P_5^*$$

(d)

$$c_{26} = G_{26} + G_{25} \cdot P_{26} + G_{24} \cdot P_{25} \cdot P_{26} +$$
$$c_{23} \cdot P_{24} \cdot P_{25} \cdot P_{26}$$