EE 2720, Spr. 2012

Homework #3 solution

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Problem 1: The right side of (T11′) is

$(X+Y)\cdot(X'+Z) = X\cdot Z + X'\cdot Y$ (1)

The left side of (T11′) is

$(X+Y)\cdot(X'+Z)\cdot(Y+Z) = (X\cdot Z + X'\cdot Y)\cdot(Y+Z)$

$= X\cdot Z\cdot Y + X\cdot Z\cdot Z + X'\cdot Y\cdot Y + X'\cdot Y\cdot Z$

$= X\cdot Y\cdot Z + X\cdot Z + X'\cdot Y + X'\cdot Y\cdot Z$

$= X\cdot Z + X'\cdot Y + X\cdot Y\cdot Z + X'\cdot Y\cdot Z$

$= X\cdot Z + X'\cdot Y + Y\cdot Z\cdot(X+X')$

$= X\cdot Z + X'\cdot Y + Y\cdot Z\cdot 1 = X\cdot Z + X'\cdot Y + Y\cdot Z$

↑ consensus term and can be eliminated according to (T11)

$= X\cdot Z + X'\cdot Y$ (2)

Because both right and left side of (T11′) reduced to the same expression (which is $X\cdot Z + X'\cdot Y$), theorem (T11′) is valid.

Problem 2: I will first provide the canonical sum and then the canonical product for each logic function.

(a) $F = \Sigma_{X,Y}(1,2) = X' \cdot Y + X \cdot Y' = \Pi_{X,Y}(0,3)$
$= (X+Y) \cdot (X'+Y')$

(b) $\Pi_{A,B}(0,1,2) = \text{minterm } 3 = A \cdot B$
$= \Pi_{A,B}(0,1,2) = (A+B) \cdot (A+B') \cdot (A'+B)$

(c) $F = \Sigma_{A,B,C}(3,4,6,7) = A' \cdot B \cdot C' + A \cdot B' \cdot C'$
$+ A \cdot B \cdot C' + A \cdot B \cdot C = \Pi_{A,B,C}(0,1,3,5)$
$= (A+B+C) \cdot (A+B+C') \cdot (A+B'+C') \cdot (A'+B+C')$

(d) $F = \Pi_{M,N,P}(0,1,3,6,7) = \Sigma_{M,N,P}(2,4,5)$
$= M' \cdot N \cdot P' + M \cdot N' \cdot P' + M \cdot N' \cdot P$
$= \Pi_{M,N,P}(0,1,3,6,7)$
$= (M+N+P) \cdot (M+N+P') \cdot (M+N'+P') \cdot$
$(M'+N'+P) \cdot (M'+N'+P')$

⇨ NEXT PAGE ⟶

Problem 2 cont: (e) $F = X + Y' \cdot Z'$

$$= X \cdot \underbrace{(Y+Y')}_{1} \cdot \underbrace{(Z+Z')}_{1} + Y' \cdot Z' \cdot \underbrace{(X+X')}_{1}$$

$$= X \cdot (Y \cdot Z + Y \cdot Z' + Y' \cdot Z + Y' \cdot Z') + Y' \cdot Z' \cdot X$$

$$+ Y' \cdot Z' \cdot X'$$

$$= \underset{1}{X} \cdot \underset{1}{Y} \cdot \underset{1}{Z} + \underset{1}{X} \cdot \underset{1}{Y} \cdot \underset{0}{Z'} + \underset{1}{X} \cdot \underset{0}{Y'} \cdot \underset{1}{Z} + \underset{1}{X} \cdot \underset{0}{Y'} \cdot \underset{0}{Z'}$$

$$+ \cancel{\underset{1}{X} \cdot \underset{0}{Y'} \cdot \underset{0}{Z'}} + \underset{0}{X'} \cdot \underset{0}{Y'} \cdot \underset{0}{Z'}$$

$$= \Sigma_{X,Y,Z}(0, 4, 5, 6, 7) = \Pi_{X,Y,Z}(1, 2, 3)$$

$$= (X + Y + Z') \cdot (X + Y' + Z) \cdot (X + Y' + Z')$$

(f) $F = A' \cdot B + B' \cdot C + A$

$$= A' \cdot B \cdot \underbrace{(C + C')}_{1} + B' \cdot C \cdot \underbrace{(A + A')}_{1}$$

$$+ A \cdot \underbrace{(B + B')}_{1} \cdot \underbrace{(C + C')}_{1}$$

NEXT PAGE →

Problem 2(f) cont:

$= A' \cdot B \cdot C + A' \cdot B \cdot C' + A \cdot B' \cdot C + A' \cdot B' \cdot C$

$+ A \cdot (B \cdot C + B \cdot C' + B' \cdot C + B' \cdot C')$

$= \underset{011}{A' \cdot B \cdot C} + \underset{010}{A' \cdot B \cdot C'} + \underset{101}{A \cdot B' \cdot C} + \underset{001}{A' \cdot B' \cdot C}$

$+ \underset{111}{A \cdot B \cdot C} + \underset{110}{A \cdot B \cdot C'} + \cancel{\underset{101}{A \cdot B' \cdot C}} + \underset{100}{A \cdot B' \cdot C'}$

$= \Sigma_{A,B,C}(1, 2, 3, 4, 5, 6, 7) = \text{maxterm } 0 =$

$A + B + C$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

<u>Problem 3:</u> $F = (a'+b)\cdot(a+c')\cdot(b'+c)$

$= (a'+b+\underbrace{c\cdot c'}_{0})\cdot(a+c'+\underbrace{b\cdot b'}_{0})\cdot(b'+c+\underbrace{a\cdot a'}_{0})$

$= (a'+b+c)\cdot(a'+b+c')\cdot(a+c'+b)\cdot(a+c'+b')$

$(b'+c+a)\cdot(b'+c+a')$

$= \underset{1\ 0\ 0}{(a'+b+c)}\cdot\underset{1\ 0\ 1}{(a'+b+c')}\cdot\underset{0\ 0\ 1}{(a+b+c')}\cdot\underset{0\ 1\ 1}{(a+b'+c')}\cdot$

$\underset{0\ 1\ 0}{(a+b'+c)}\cdot\underset{1\ 1\ 0}{(a'+b'+c)}$

$= \prod_{a,b,c}(1,2,3,4,5,6).$

Problem 4: (a) Algebraic approach:

$$F = a' \cdot b + c \cdot d' + e$$
$$= [(a' \cdot b + c \cdot d' + e)']'$$
$$= [(a' \cdot b)' \cdot (c \cdot d')' \cdot e']' \quad (1)$$

From (1) we get the following figure

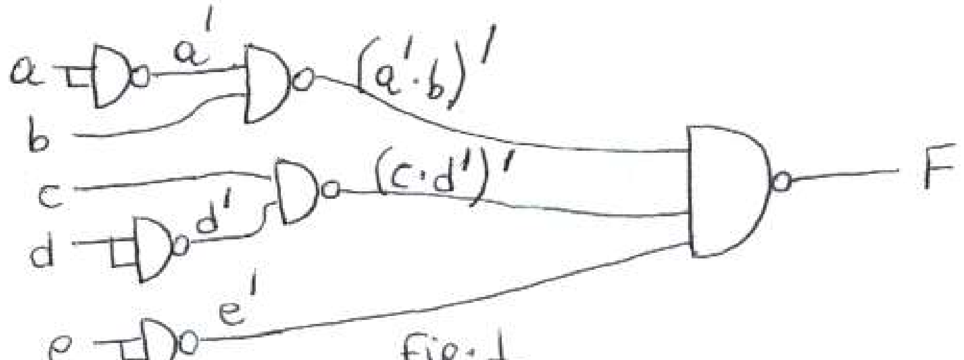

fig. 1

(b) Graphical approach: I first provide a logic circuit showing an AND-OR realization of F

(Figure on next page)

Prob. 4 cont

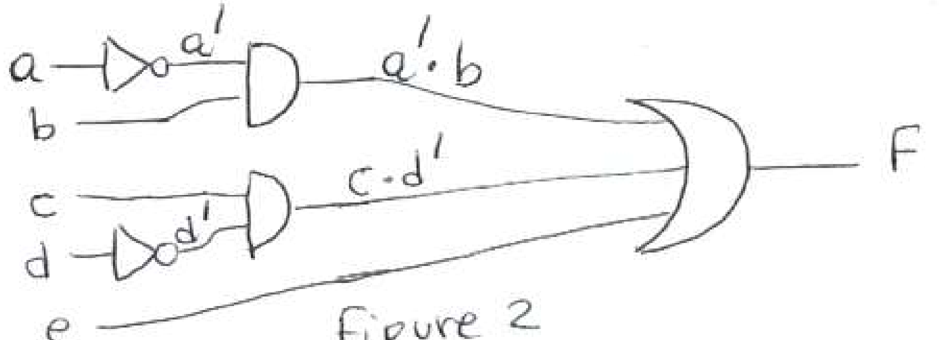

Figure 2

From the above fig. 2 one gets

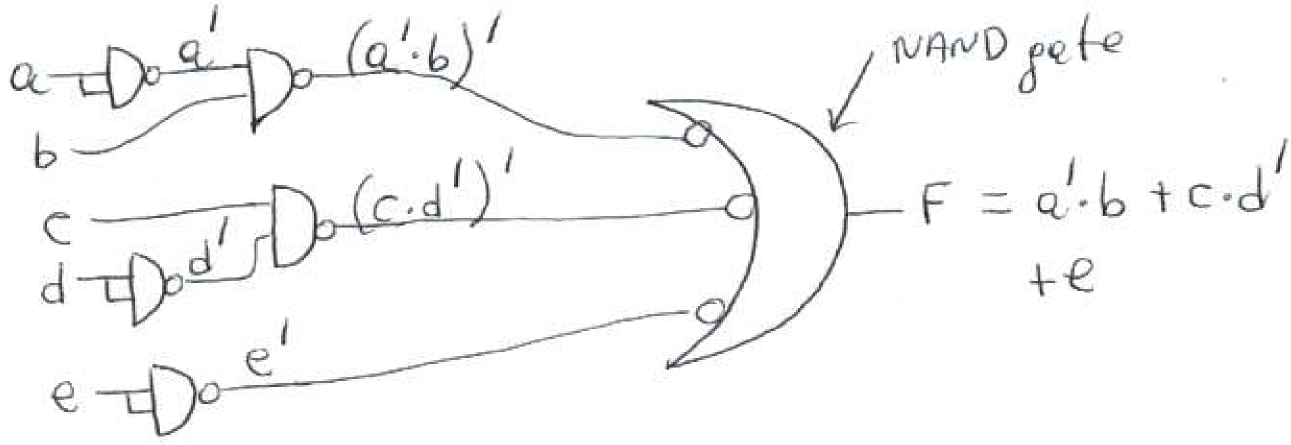

Figure 3

Problem 5: (a) Algebraic approach

$$F = (a'+b)\cdot(c+d')\cdot e$$
$$= \left[\left[(a'+b)\cdot(c+d')\cdot e\right]'\right]'$$
$$= \left[(a'+b)' + (c+d')' + e'\right]' \quad (2)$$

From (2) we get the following figure

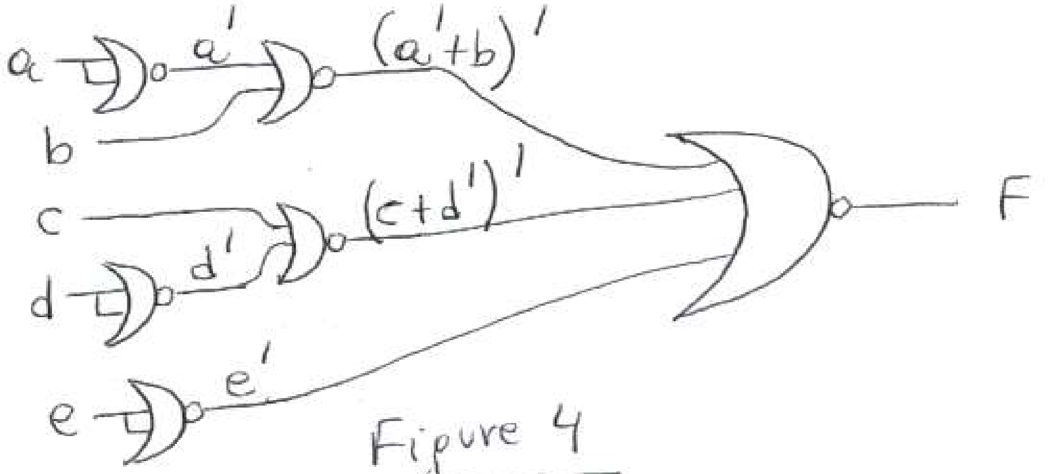

Figure 4

(b) Graphical approach: I first provide a logic circuit showing an OR-AND realization of F. This is shown on the next page.

Prob. 5 cont.

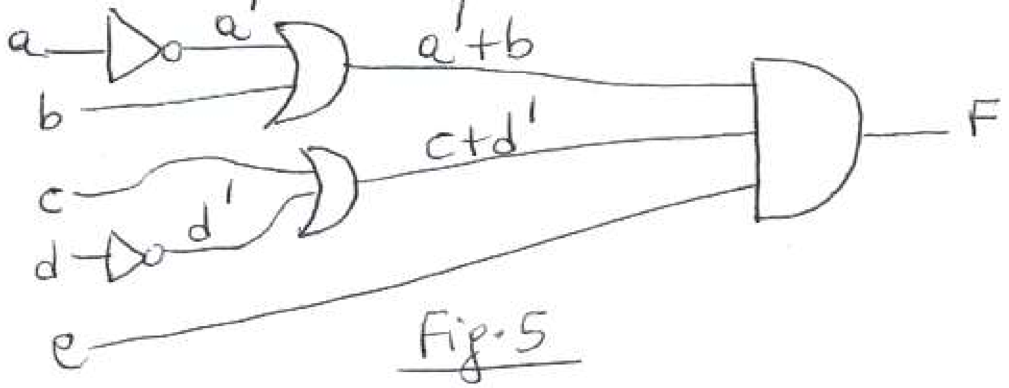

Fig. 5

From the above figure 5 one gets

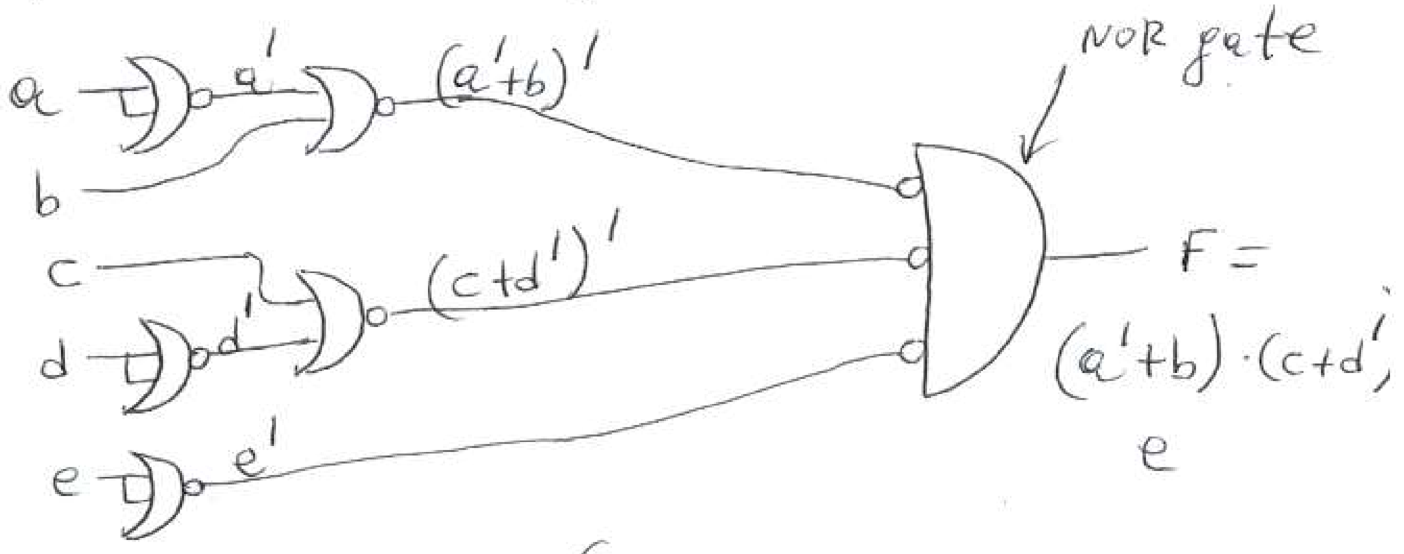

Figure 6

EE 2720 HW #3 solutions cont.

Problem 6:

(a) Proof of (5) or proof of $X \oplus 0 = X$

- Case $X = 0$: $0 \oplus 0 = 0$
- Case $X = 1$: $1 \oplus 0 = 1$

(b) Proof of (6) or proof of $X \oplus 1 = X'$

- Case $X = 0$: $0 \oplus 1 = 1$
- Case $X = 1$: $1 \oplus 1 = 0$

(c) Proof of (7) or proof of $X \oplus X = 0$

- Case $X = 0$: $0 \oplus 0 = 0$
- Case $X = 1$: $1 \oplus 1 = 0$

(d) Proof of (8) or proof of $X \oplus X' = 1$

- Case $X = 0$: $0 \oplus 1 = 1$
- Case $X = 1$: $1 \oplus 0 = 1$

Note: The above eqs. (5) – (8) can also be proven algebraically. How?

(e) Proof of (11) or proof of

$$X \cdot (Y \oplus Z) = X \cdot Y \oplus X \cdot Z$$

NEXT PAGE →

EE 2720, HW#3 solutions cont.

Problem 6 (e) cont:

The right side of (11) is:

$$X \cdot Y \oplus X \cdot Z = X \cdot Y \cdot (X \cdot Z)' + (X \cdot Y)' \cdot X \cdot Z$$

$$= X \cdot Y \cdot (X' + Z') + (X' + Y') \cdot X \cdot Z$$

$$= \underbrace{X \cdot Y \cdot X'}_{0} + X \cdot Y \cdot Z' + \underbrace{X' \cdot X \cdot Z}_{0} + Y' \cdot X \cdot Z$$

$$= X \cdot Y \cdot Z' + X \cdot Y' \cdot Z = X \cdot (Y \cdot Z' + Y' \cdot Z)$$

$$= X \cdot (Y \oplus Z) = \text{left side of (11)} \Rightarrow \text{proven.}$$