EE 2720, Spring 2012

Homework #2 solution

**Problem 1:**

```
  010100 ← positive number
+)001111 ← positive number
 0 100011 ← negative number
```

carry out to be ignored.

↳ sign bit = 1 ⇒ we got a negative result. Here an overflow occured. Just see that the Dynamic Range (DR) of a 6-bit two's-complement system is $DR = [-2^5 \quad 2^5 - 1] = [-32 \quad +31]$ and $X+Y = +20+15 = +35 > 31$ which ⇒ overflow.

**Problem 2:**

```
  101100 ← negative number
+)110001 ← negative   "
 1 011101 ← positive result
```

Ignore carry out

↳ sign bit = 0 ⇒ positive result. Here an underflow occured. As said in problem 1 the Dynamic Range (DR) is $[-32 \quad 31]$ and $X+Y = (-20)+(-15) = -35 < -32$ ⇒ underflow.

**Problem 3:** Here the two numbers are of different signs and we therefore have to perform magnitude of X − magnitude of Y

$= (10100) - (11110)$

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$= (10100) + (\text{two's-compl. of } (11110))$

$$
\begin{array}{rr}
= & 10100 \\
+) & 00010 \\
\hline
 & 010110
\end{array}
$$

$\hookrightarrow c = 0 \Rightarrow \text{result} < 0 \Rightarrow$

magnitude of X − magn. of Y < 0 ⇒

magn. of X < magn. of Y

Thus

- sign bit of X+Y = sign bit of Y = 1

  and magnitude of X+Y

$= \text{two's compl. of } (10110) = 01010$

Thus $X+Y = 101010_2 = -10_{10}$

**Problem 4:**

$$
\begin{array}{rl}
1101 & \text{multiplicand} \\
1110 & \text{multiplier} \\
\hline
0000 & \\
+)\ 1101\phantom{0} & \\
\hline
11010 & \\
+)\ 1101\phantom{00} & \\
\hline
1001110 & \\
+)\ 1101\phantom{000} & \\
\hline
10110110 &
\end{array}
$$

→ product = 182

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# EE 2720 HW#2 sol. cont.

**Problem 5:** I don't provide solution since you were not supposed to do this problem 5.

**Problem 6:**

$$
\begin{array}{r}
7 \\
+\ 9 \\
\hline
16
\end{array}
\qquad\qquad
\begin{array}{rl}
0111 & \\
+)\ 1001 & \\
\hline
10000 & \leftarrow \text{correction needed} \\
+)\ \ 110 & \\
\hline
\underbrace{1}_{1}\,\underbrace{0110}_{6} &
\end{array}
$$

**Problem 7:**

$$
\begin{array}{r}
5 \\
+4 \\
\hline
9
\end{array}
\qquad\qquad
\begin{array}{rl}
0101 & \\
+)\ 0100 & \\
\hline
\underbrace{1001}_{9} & \leftarrow \text{no correction needed}
\end{array}
$$

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Problem 8: All theorems are one-variable theorems. Just view two cases: $X=0$ and $X=1$ and apply the axioms. Proofs are trivial. See notes # 5 for similar proofs.

Problem 9:

| X Y Z | X+Y | Y+Z | (X+Y)+Z | X+(Y+Z) |
|---|---|---|---|---|
| 0 0 0 | 0 | 0 | 0 | 0 |
| 0 0 1 | 0 | 1 | 1 | 1 |
| 0 1 0 | 1 | 1 | 1 | 1 |
| 0 1 1 | 1 | 1 | 1 | 1 |
| 1 0 0 | 1 | 0 | 1 | 1 |
| 1 0 1 | 1 | 1 | 1 | 1 |
| 1 1 0 | 1 | 1 | 1 | 1 |
| 1 1 1 | 1 | 1 | 1 | 1 |

same identical columns

$\Rightarrow (X+Y)+Z = X+(Y+Z)$

and the theorem is proven.

# EE 2720, HW # 2 solutions



**Problem 10:** (T10') states:

$$(X+Y)\cdot(X+Y')=X$$

**Proof:** $(X+Y)\cdot(X+Y') = X\cdot X + X\cdot Y' + Y\cdot X + Y\cdot Y'$
$= X + X\cdot Y' + X\cdot Y + 0 = X + X\cdot Y' + X\cdot Y =$
$X\cdot(1+Y'+Y) = X\cdot 1 = X$

**Problem 11:** (T13') states:
$$(X_1 + X_2 + \cdots + X_n)' = X_1'\cdot X_2' \cdot \ldots \cdot X_n'$$

**Proof:** I'll first prove (T13') for $n=2$ or I'll prove that $(X_1+X_2)' = X_1'\cdot X_2'$ (1)

I'll prove (1) using a truth table

| $X_1 X_2$ | $X_1+X_2$ | $(X_1+X_2)'$ | $X_1'$ | $X_2'$ | $X_1'\cdot X_2'$ |
|---|---|---|---|---|---|
| 0 0 | 0 | 1 | 1 | 1 | 1 |
| 0 1 | 1 | 0 | 1 | 0 | 0 |
| 1 0 | 1 | 0 | 0 | 1 | 0 |
| 1 1 | 1 | 0 | 0 | 0 | 0 |

identical column $\Rightarrow$
eq. (1) true.

Problem 11 cont. Assume now that theorem (T13') is true for $n=i$, or assume that

$$(X_1+X_2+\cdots+X_i)' = X_1'\cdot X_2'\cdots X_i' \quad (2)$$

We need to prove that the theorem is also true for $n=i+1$ or we need to prove that

$$(X_1+X_2+\cdots X_i+X_{i+1})' = X_1'\cdot X_2'\cdots X_i'\cdot X_{i+1}'$$

But $(X_1+X_2+\cdots X_i+X_{i+1})'$

$$= [(X_1+X_2+\cdots+X_i)+X_{i+1}]'$$

$$= (X_1+X_2+\cdots+X_i)'\cdot X_{i+1}' \quad \text{(according to (1))}$$

$$= X_1'\cdot X_2'\cdots X_i'\cdot X_{i+1}' \quad \text{(according to (2))}$$

The proof is now completed.

Problem 12: $(X+Y)\cdot(X'+Z) = X\cdot X' + X\cdot Z + Y\cdot X' + Y\cdot Z = 0 + X\cdot Z + X'\cdot Y + Y\cdot Z$

$= X\cdot Z + X'\cdot Y + Y\cdot Z$ consensus term and can be eliminated according to (T11)

$= X\cdot Z + X'\cdot Y \Rightarrow$ proven.

Problem 13: The left side of (T10) is

$X \cdot Y + X \cdot Y'$. This can be written as

$Y \cdot X + Y' \cdot X = Y \cdot X + Y' \cdot X + \underbrace{X \cdot X}_{\text{consensus term}}$ (according to (T11))

$= Y \cdot X + Y' \cdot X + X = X \cdot (Y + Y' + 1) = X \cdot 1 = X$. we now reached the right side of (T10) so the proof is completed

Problem 14:

a) $F = W \cdot X \cdot Y \cdot Z \, (W \cdot X \cdot Y \cdot Z' + W \cdot X' \cdot Y \cdot Z + W' \cdot X \cdot Y \cdot Z + W \cdot X \cdot Y' \cdot Z) = W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y \cdot Z'$
$+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X' \cdot Y \cdot Z + W \cdot X \cdot Y \cdot Z \cdot W' \cdot X \cdot Y \cdot Z$
$+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y' \cdot Z = 0 + 0 + 0 + 0 = 0$.

b) $F = A \cdot B + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E + C' \cdot D \cdot E$

$= A \cdot B \cdot (1 + C' D + D \cdot E' + C' \cdot E) + C' \cdot D \cdot E = A \cdot B \cdot 1 + C' \cdot D \cdot E = A \cdot B + C' \cdot D \cdot E$