

EE 2720, Spring 2012  
Homework #2 solution.

Problem 1:

$$\begin{array}{r} 010100 \leftarrow \text{positive number} \\ +) 001111 \leftarrow \text{positive number} \\ \hline 0100011 \leftarrow \text{negative number} \end{array}$$

Copy out  
to be ignored.

↳ sign bit = 1  $\Rightarrow$  we got a negative result. Here an overflow occurred. Just see that

the Dynamic Range (DR) of a 6-bit two's-complement system is  $DR = [-2^5, 2^5 - 1] = [-32, 31]$  and  $X+Y = +20+15 = +35 > 31$  which  $\Rightarrow$  overflow.

Problem 2:  $101100 \leftarrow$  negative number

$$\begin{array}{r} +) 110001 \leftarrow \text{negative " } \\ \hline 101101 \leftarrow \text{positive result} \end{array}$$

Ignore  
copy out

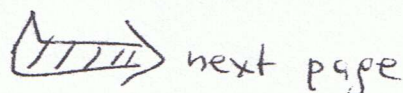
↳ sign bit = 0  $\Rightarrow$  positive result. Here an underflow occurred. As said

in problem 1 the Dynamic Range (DR) is  $[-32, 31]$  and  $X+Y = (-20)+(-15) = -35 < -32 \Rightarrow$  underflow.

Problem 3: Here the two numbers are of

different signs and we therefore have to perform magnitude of X - magnitude of Y

$$= (10100) - (11110)$$

 next page

$$= (10100) + (\text{two's compl. of } (11110))$$

$$= \begin{array}{r} 10100 \\ +) 00010 \\ \hline 010110 \end{array}$$

$\hookrightarrow c=0 \Rightarrow \text{result} < 0 \Rightarrow$

magn. of  $X$  - magn. of  $Y < 0 \Rightarrow$

magn. of  $X < \text{magn. of } Y$

Thus

- sign bit of  $X+Y = \text{sign bit of } Y = 1$   
and magn. of  $X+Y$

$$= \text{two's compl. of } (10110) = 01010$$

$$\text{Thus } X+Y = 101010_2 = -10_{10}$$

Problem 4:

$$\begin{array}{r} 1101 \text{ multiplicand} \\ 1110 \text{ multiplier} \\ \hline 0000 \\ +) 1101 \\ \hline 0101 \\ +) 1101 \\ \hline 100110 \\ +) 1101 \\ \hline 1011010 \end{array}$$

$\rightarrow \text{product} = 182$

$\Rightarrow$  next page

Problem 5: I don't provide solution since you were not supposed to do this problem 5.

Problem 6:

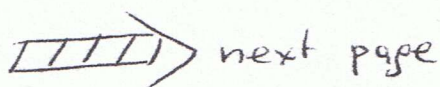
$$\begin{array}{r} 7 \\ + 9 \\ \hline 16 \end{array}$$

$$\begin{array}{r} 0111 \\ +) 1001 \\ \hline 10000 \leftarrow \text{correction needed} \\ +) 110 \\ \hline \underline{10110} \\ 16 \end{array}$$

Problem 7:

$$\begin{array}{r} 5 \\ + 4 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 0101 \\ +) 0100 \\ \hline 1001 \leftarrow \text{no correction needed} \\ \hline 9 \end{array}$$

 next page

Problem 8: All theorems are one-variable theorems. Just view two cases:  $\begin{cases} \rightarrow X=0 \\ \rightarrow X=1 \end{cases}$  and apply the axioms. Proofs are trivial. See notes # 5 for similar proofs.

Problem 9:

X	Y	Z	X+Y	Y+Z	(X+Y)+Z	X+(Y+Z)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1



same identical columns  
 $\Rightarrow (X+Y)+Z = X+(Y+Z)$   
 and the theorem is proven.

Problem 10: (T10') states:

$$(X+Y) \cdot (X+Y') = X$$

Proof:  $(X+Y) \cdot (X+Y') = X \cdot X + X \cdot Y' + Y \cdot X + Y \cdot Y'$   
 $= X + X \cdot Y' + X \cdot Y + 0 = X + X \cdot Y' + X \cdot Y =$   
 $X \cdot (1 + Y' + Y) = X \cdot 1 = X$

Problem 11: (T13') states:

$$(X_1 + X_2 + \dots + X_n)' = X_1' \cdot X_2' \cdot \dots \cdot X_n'$$

Proof: I'll first prove (T13') for  $n=2$   
 or I'll prove that  $(X_1 + X_2)' = X_1' \cdot X_2'$  (1)  
 I'll prove (1) using a truth table

$X_1 X_2$	$X_1 + X_2$	$(X_1 + X_2)'$	$X_1'$	$X_2'$	$X_1' \cdot X_2'$
00	0	1	1	1	1
01	1	0	1	0	0
10	1	0	0	1	0
11	1	0	0	0	0

↑  
 identical columns  $\Rightarrow$   
 eq. (1) true.

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Problem 11 cont. Assume now that theorem

(T13') is true for  $n=i$ , or assume that  
 $(X_1 + X_2 + \dots + X_i)' = X_1' \cdot X_2' \cdot \dots \cdot X_i'$  (2)

We need to prove that the theorem is also true for  $n=i+1$  or we need to prove that

$$(X_1 + X_2 + \dots + X_i + X_{i+1})' = X_1' \cdot X_2' \cdot \dots \cdot X_i' \cdot X_{i+1}'$$

But  $(X_1 + X_2 + \dots + X_i + X_{i+1})'$

$$= [(X_1 + X_2 + \dots + X_i) + X_{i+1}]'$$

$$= (X_1 + X_2 + \dots + X_i)' \cdot X_{i+1}' \quad (\text{according to (1)})$$

$$= X_1' \cdot X_2' \cdot \dots \cdot X_i' \cdot X_{i+1}' \quad (\text{according to (2)})$$

The proof is now completed.

Problem 12:  $(X+Y) \cdot (X'+Z) = X \cdot X' + X \cdot Z$

$$+ Y \cdot X' + Y \cdot Z = 0 + X \cdot Z + X' \cdot Y + Y \cdot Z$$

$$= X \cdot Z + X' \cdot Y + Y \cdot Z$$

$\swarrow$  consensus term and can be eliminated according to (T11)

$$= X \cdot Z + X' \cdot Y \Rightarrow \text{proven.}$$

Problem 13: The left side of (T10) is

$X \cdot Y + X \cdot Y'$ . This can be written as

$$Y \cdot X + Y' \cdot X = Y \cdot X + Y' \cdot X + \underbrace{X \cdot X}_{\text{consensus term}}$$

$$= Y \cdot X + Y' \cdot X + X = X \cdot (Y + Y' + 1) = X \cdot 1 = X$$

we now reached the right side of (T10) so the proof is completed

Problem 14:

$$\begin{aligned} \text{a) } F &= W \cdot X \cdot Y \cdot Z (W \cdot X \cdot Y \cdot Z' + W \cdot X' \cdot Y \cdot Z + W \cdot X \cdot Y' \cdot Z \\ &+ W \cdot X \cdot Y \cdot Z) = W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y \cdot Z' \\ &+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X' \cdot Y \cdot Z + W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y' \cdot Z \\ &+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y \cdot Z = 0 + 0 + 0 + 0 = 0 \end{aligned}$$

$$\begin{aligned} \text{b) } F &= A \cdot B + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E \\ &+ C' \cdot D \cdot E \\ &= A \cdot B \cdot (1 + C' \cdot D + D \cdot E' + C' \cdot E) + C' \cdot D \cdot E = A \cdot B \cdot 1 \\ &+ C' \cdot D \cdot E = A \cdot B + C' \cdot D \cdot E \end{aligned}$$