EE 2720

Handout # 7

One of the subjects studied in handout # 6 was the subject of <u>multiplying out</u> and <u>factoring</u>. As we said, by multiplying out we mean transforming an expression from product-of-sums form into the corresponding sum-of-products form and by factoring we mean transforming an expression from sum-of-products form into the corresponding product-of-sums-form. The <u>usefulness</u> of multiplying out and factoring is that you can transform an <u>OR-AND</u> logic circuit into an <u>AND-OR</u> logic circuit and vice versa.

To be more specific:

- By multiplying out you can transform an OR-AND logic circuit into its corresponding equivalent AND-OR logic circuit.
- By factoring you can transform an AND-OR logic circuit into its corresponding equivalent OR-AND logic circuit.

I will now show you two examples, one of multiplying out and one of factoring together with their corresponding figures.

<u>Example of multiplying out:</u> Multiply out $(A+B\cdot C)\cdot(A+D+E)$

<u>Answer:</u>

$$(A+B\cdot C)\cdot(A+D+E) = A+B\cdot C\cdot D+B\cdot C\cdot E \quad (1)$$

This is the example at the top of page 5 of handout # 6.

The left hand side of equation (1) is a product-of-sums form and can be represented by an OR-AND circuit. This circuit is shown by figure 1 below. The right hand side of equation (1) is a sum-of-products form and can be represented by an AND-OR logic circuit. This circuit is shown by figure 2 below. The circuits of figures 1 and 2, although they look different, they do the same thing; (they are equivalent). Figures 1 and 2 follow

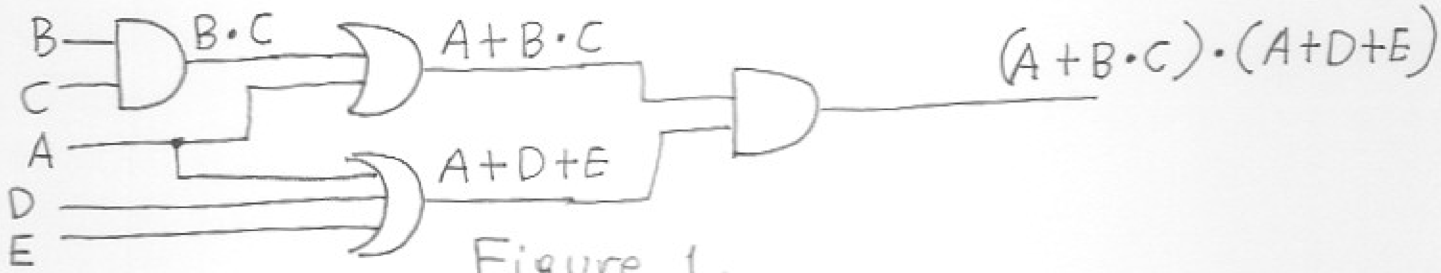


Figure 1.

Figure 2; (it is equivalent to Figure 1).

Example of factoring: Factor $A \cdot B' + C' \cdot D$

Answer:

$$A \cdot B' + C' \cdot D = (C' + A) \cdot (C' + B') \cdot (D + A) \cdot (D + B') \quad (2)$$

This is the second example of factoring on page 7 of handout # 6.

The left hand side of equation (2) is a sum-of-products form and can be represented by an AND-OR logic circuit. This circuit is shown by figure 3 below. The right hand side of equation (2) is a product-of-sums form and can be represented by an OR-AND logic circuit. This circuit is shown by figure 4 below. The circuits of figures 3 and 4, although they look different, they do the same thing; (they are equivalent). The figures 3 and 4 follow.

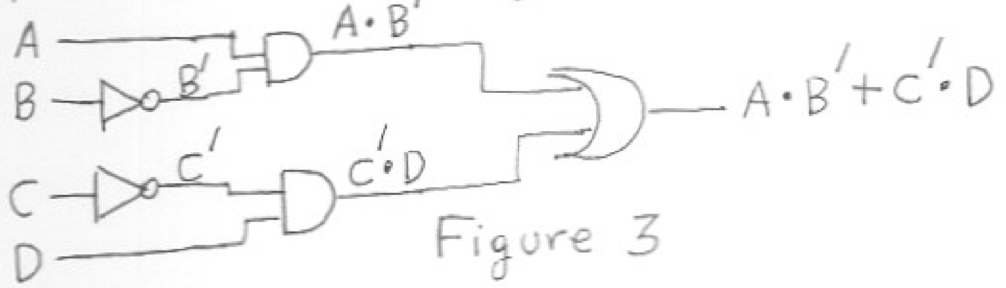


Figure 3

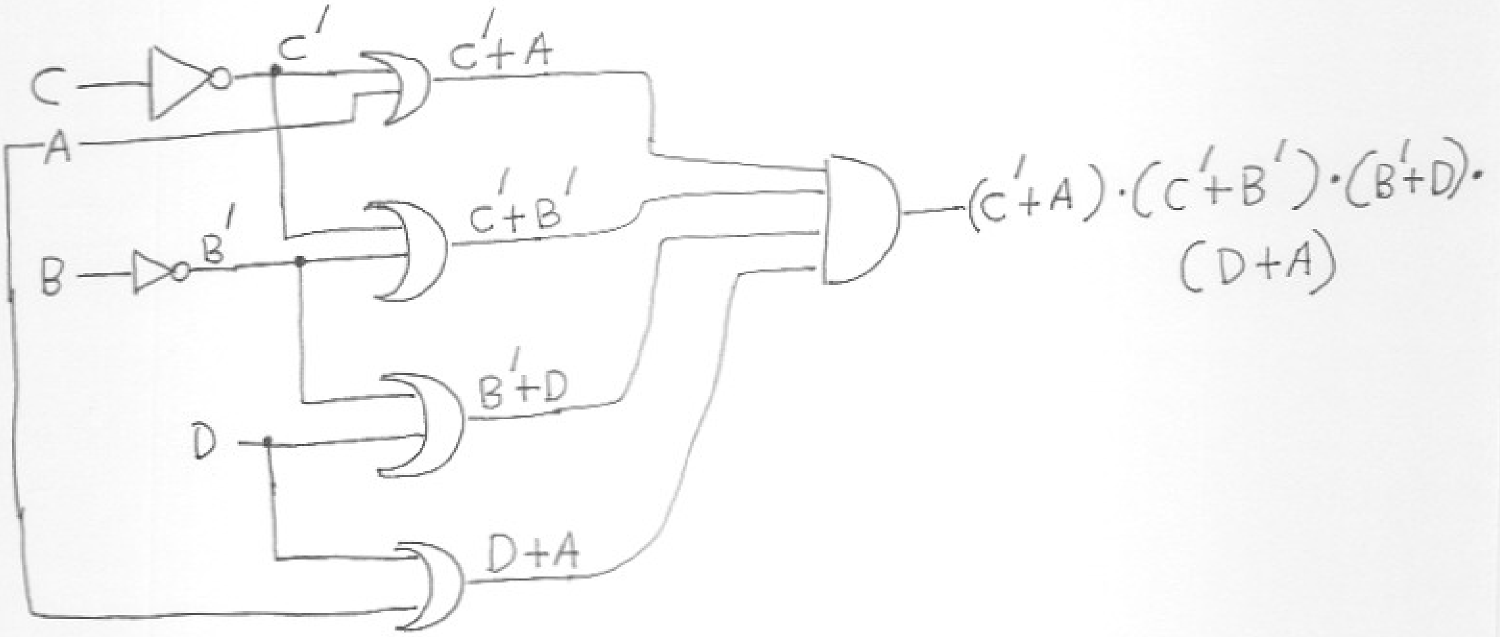


Figure 4; (it is equivalent to Fig. 3).

- <u>Proving the validity of an equation</u>

Often we need to determine if an equation is valid for all combinations of values of the variables. <u>The usefulness of this is whenever we want to determine</u>

if two logic circuits perform the same function; (if they are equivalent).

Several methods can be used to determine if an equation is valid:

1. Construct a truth table and evaluate both sides of the equation for all combinations of values of the variables. (This method is not elegant and is time consuming if the number of variables is large).
2. Manipulate one side of the equation by applying the with the other side.
3. Reduce both sides of the equation independently to the same expression.
4. It is permissible to perform the same operation on both sides of the equation provided that this operation is reversible. For example, it is OK to complement both sides of the equation, but it is <u>not</u> permissible to multiply (AND) both sides of the equation by the same expression. (Multiplication is not reversible because division is not defined for switching algebra). Likewise, it is <u>not</u> permissible to add (OR) the same term to both sides of an equation because subtraction is not defined for switching algebra.

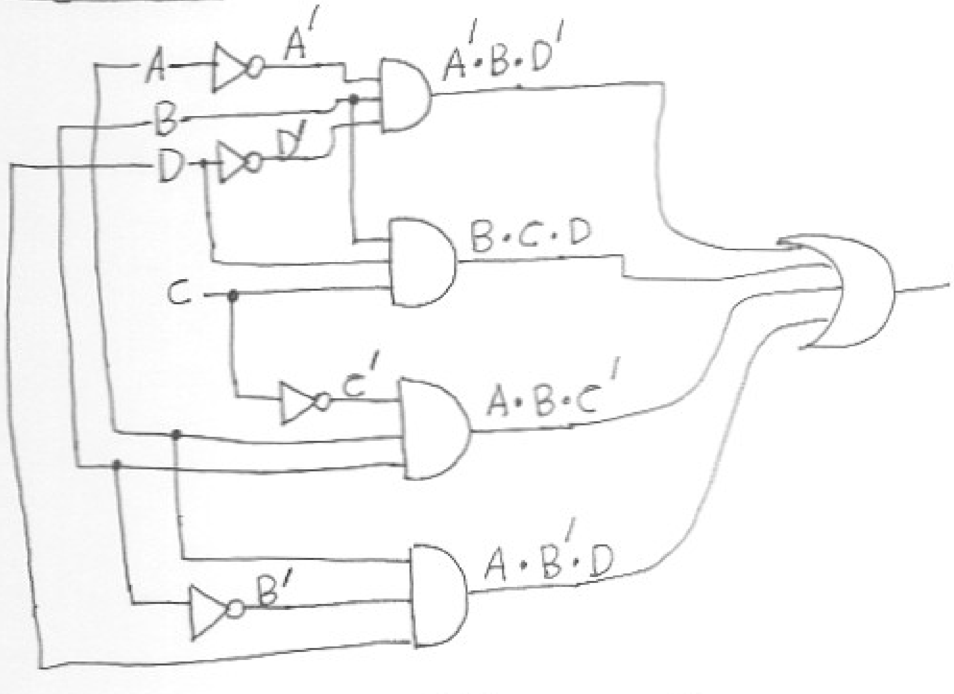
• To prove that an equation is <u>not</u> valid, it is enough to show one combination of values of the variables for which the two sides of the equation have different values.

• When using methods 2 or 3 above for proving that an equation is valid, useful things to do are:

1. First reduce both sides to a sum-of-products (or product-of-sums).

2. Compare the two sides of the equation to see how they differ.
3. Then try to add terms to one side of the equation that are present on the other side.
4. Finally try to eliminate terms from one side that are not present on the other.

Example: Consider the two logic circuits of the figures 5 and 6 which are shown below. Prove that these two logic circuits perform the same function; (they are equivalent). The figures 5 and 6 follow



$Y = A' \cdot B \cdot D' + B \cdot C \cdot D + A \cdot B \cdot C' + A \cdot B' \cdot D$

Figure 5

Figure 6 is shown on the next page.

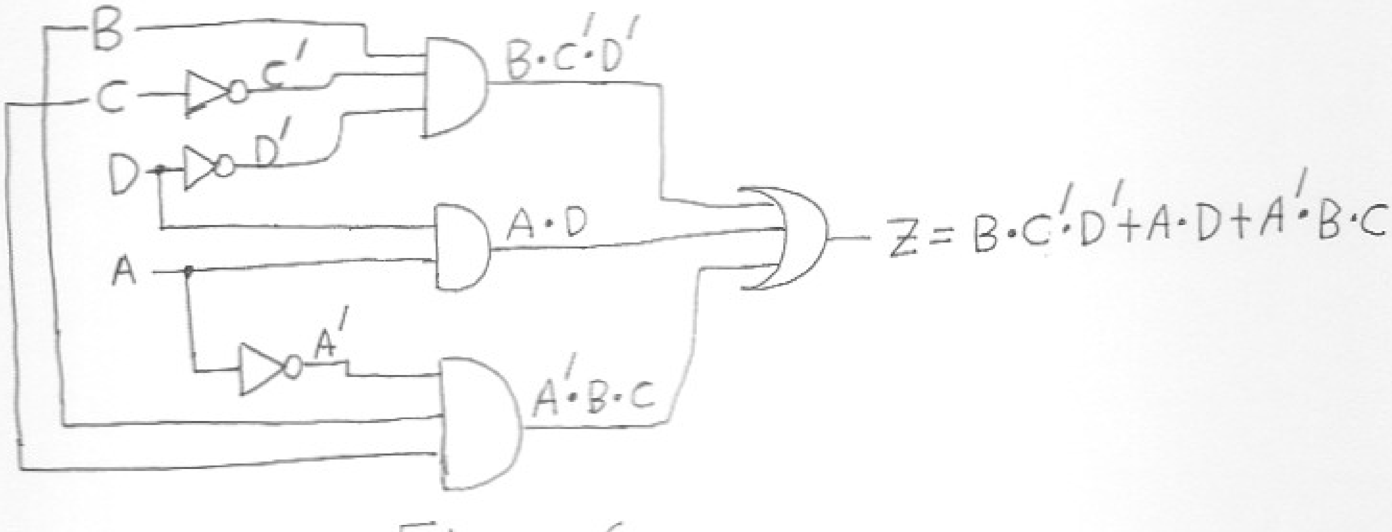


Figure 6

Answer: The logic circuits of figures 5 and 6 have the same inputs A, B, C, D. All we need to prove is that their outputs are equal or we need to prove that $Y = Z$ or prove that

$$A' \cdot B \cdot D' + B \cdot C \cdot D + A \cdot B \cdot C' + A \cdot B' \cdot D = B \cdot C' \cdot D' + A \cdot D + A' \cdot B \cdot C$$

Proof: Here we will start with the left side of the above equation, manipulate it until we reach the right side of the equation. We will apply the consensus theorem (T11) which states $X \cdot Y + X' \cdot Z + Y \cdot Z = X \cdot Y + X' \cdot Z$; ($Y \cdot Z$ is the consensus term). What we will do is first add consensus terms, then combine terms, and finally eliminate terms by the consensus theorem. We now have

$\underline{A' \cdot B \cdot D'} + B \cdot C \cdot D + \underline{A \cdot B \cdot C'} + A \cdot B' \cdot D$ (add consensus of $A' \cdot B \cdot D'$ and $A \cdot B \cdot C'$)

$= A' \cdot B \cdot D' + B \cdot C \cdot D + A \cdot B \cdot C' + A \cdot B' \cdot D + B \cdot C' \cdot D'$ (add consensus of $A' \cdot B \cdot D'$ and $B \cdot C \cdot D$)

$= A' \cdot B \cdot D' + B \cdot C \cdot D + A \cdot B \cdot C' + A \cdot B' \cdot D + B \cdot C' \cdot D' + A' \cdot B \cdot C$ (add consensus of $B \cdot C \cdot D$ and $A \cdot B \cdot C'$)

$= A'\cdot B\cdot D' + B\cdot C\cdot D + A\cdot B\cdot C' + A\cdot B'\cdot D + B\cdot C'\cdot D' + A'\cdot B\cdot C + A\cdot B\cdot D$

$= A\cdot B\cdot D + A\cdot B'\cdot D + A'\cdot B\cdot D' + B\cdot C\cdot D + A\cdot B\cdot C' + B\cdot C'\cdot D' + A'\cdot B\cdot C$

$= A\cdot D\cdot(B+B') + A'\cdot B\cdot D' + B\cdot C\cdot D + A\cdot B\cdot C' + B\cdot C'\cdot D' + A'\cdot B\cdot C =$

$= A\cdot D\cdot 1 + A'\cdot B\cdot D' + B\cdot C\cdot D + A\cdot B\cdot C' + B\cdot C'\cdot D' + A'\cdot B\cdot C =$

$= A\cdot D + A'\cdot B\cdot D' + B\cdot C\cdot D + A\cdot B\cdot C' + B\cdot C'\cdot D' + A'\cdot B\cdot C$ (apply consensus theorem on terms $A\cdot D$, $B\cdot C'\cdot D'$, $A\cdot B\cdot C'$ to eliminate $A\cdot B\cdot C'$)

$= A\cdot D + A'\cdot B\cdot D' + B\cdot C\cdot D + B\cdot C'\cdot D' + A'\cdot B\cdot C$ (apply consensus theorem on terms $A\cdot D$, $A'\cdot B\cdot C$, $B\cdot C\cdot D$ to eliminate $B\cdot C\cdot D$)

$= A\cdot D + A'\cdot B\cdot D' + B\cdot C'\cdot D' + A'\cdot B\cdot C$ (apply consensus theorem on terms $B\cdot C'\cdot D'$, $A'\cdot B\cdot C$, $A'\cdot B\cdot D'$ to eliminate $A'\cdot B\cdot D'$)

$= A\cdot D + B\cdot C'\cdot D' + A'\cdot B\cdot C = B\cdot C'\cdot D' + A\cdot D + A'\cdot B\cdot C$.

We have now reached the right side of the equation and the proof is completed.

<u>Example</u>: In this example I do not provide any figure. Show that the following equation is valid:

$A'\cdot B\cdot C'\cdot D + (A'+B\cdot C)\cdot(A+C'\cdot D') + B\cdot C'\cdot D + A'\cdot B\cdot C' =$

$= A\cdot B\cdot C\cdot D + A'\cdot C'\cdot D' + A\cdot B\cdot D + A\cdot B\cdot C\cdot D' + B\cdot C'\cdot D$

<u>Answer</u>: The method we will use here is to reduce both sides of the equation independently to the same expression. The basic theorems to be used will be the consensus theorem (T11) and the theorem that states $(X+Y)\cdot(X'+Z) = X\cdot Z + X'\cdot Y$ (3). This theorem of equation (3) was stated and proved on

page 4 of handout #6. We will first reduce the left hand side of the given equation. The left hand side is

$A'\cdot B\cdot C'\cdot D+(A'+B\cdot C)\cdot(A+C'\cdot D')+B\cdot C'\cdot D+A'\cdot B\cdot C'=$
$=(A'+B\cdot C)\cdot(A+C'\cdot D')+B\cdot C'\cdot D+A'\cdot B\cdot C'\cdot D+A'\cdot B\cdot C'\cdot 1=$
$=(A'+B\cdot C)\cdot(A+C'\cdot D')+B\cdot C'\cdot D+A'\cdot B\cdot C'\cdot(D+1)=$
$=(A'+B\cdot C)\cdot(A+C'\cdot D')+B\cdot C'\cdot D+A'\cdot B\cdot C'\cdot 1=$
$=(A'+B\cdot C)\cdot(A+C'\cdot D')+B\cdot C'\cdot D+A'\cdot B\cdot C'$ (apply theorem of equation (3) of previous page)
$=A\cdot B\cdot C+A'\cdot C'\cdot D'+B\cdot C'\cdot D+A'\cdot B\cdot C'$ (apply consensus theorem on terms $A'\cdot C'\cdot D'$, $B\cdot C'\cdot D$, $A'\cdot B\cdot C'$ to eliminate $A'\cdot B\cdot C'$)
$=A\cdot B\cdot C+A'\cdot C'\cdot D'+B\cdot C'\cdot D$

The above expression $A\cdot B\cdot C+A'\cdot C'\cdot D'+B\cdot C'\cdot D$ is the result of reducing the left hand side of the original given equation.

We will now reduce the right hand side of the equation. The right hand side is

$A\cdot B\cdot C\cdot D+A'\cdot C'\cdot D'+A\cdot B\cdot D+A\cdot B\cdot C\cdot D'+B\cdot C'\cdot D=$
$=A\cdot B\cdot C\cdot D+A\cdot B\cdot C\cdot D'+A'\cdot C'\cdot D'+A\cdot B\cdot D+B\cdot C'\cdot D=$
$=A\cdot B\cdot C\cdot(D+D')+A'\cdot C'\cdot D'+A\cdot B\cdot D+B\cdot C'\cdot D=$
$=A\cdot B\cdot C\cdot 1+A'\cdot C'\cdot D'+A\cdot B\cdot D+B\cdot C'\cdot D=$
$=A\cdot B\cdot C+A'\cdot C'\cdot D'+A\cdot B\cdot D+B\cdot C'\cdot D$ (apply consensus theorem on terms $A\cdot B\cdot C$, $B\cdot C'\cdot D$, $A\cdot B\cdot D$ to eliminate $A\cdot B\cdot D$)
$=A\cdot B\cdot C+A'\cdot C'\cdot D'+B\cdot C'\cdot D$

The above expression $A\cdot B\cdot C+A'\cdot C'\cdot D'+B\cdot C'\cdot D$ is the result of reducing the right hand side of the original given equation.

Because both sides (left and right) of the original equation were independently reduced to the same expression, the original equation is valid.

Important Note: Some of the theorems of switching algebra are not true for ordinary algebra. Similarly, some of the theorems of ordinary algebra are not true for switching algebra. Consider for example the cancellation law for ordinary algebra

$$\text{If } X+Y=X+Z \text{ then } Y=Z$$

The above cancellation law is not true for switching algebra. I will demonstrate this by creating an example in which $X+Y=X+Z$ but $Y\neq Z$. Let $X=1$, $Y=0$, $Z=1$. Then $1+0=1+1=1$ but $0\neq 1$; ($+$ means OR here).

In ordinary algebra, the cancellation law for multiplication is

$$\text{If } X\cdot Y=X\cdot Z \text{ then } Y=Z.$$

The above law is valid provided that $X\neq 0$.
In switching algebra, the cancellation law for logical multiplication is also not valid when $X=0$. (Let $X=0$, $Y=0$, $Z=1$; then $0\cdot 0=0\cdot 1=0$ but $0\neq 1$).
Because $X=0$ about half of the time in switching algebra, the cancellation law for logical multiplication (AND operation) is not true for switching algebra.

The conclusion is that you are not allowed to use the cancellation laws for OR and AND operations in switching algebra when you try to prove the validity of an equation.