EE 2720

Handout #11

• <u>Transforming an AND-OR logic circuit into an OR-AND logic circuit and vice versa.</u>

The above subject was studied in handout# 7. To remind you:

• <u>Transforming an AND-OR logic circuit into an OR-AND logic circuit.</u>

<u>Do the following</u>: Draw the figure of the AND-OR logic circuit. Compute its output F. F is going to be in sum-of-products form. Transform F into product-of-sums form by factoring for example; (what other algebraic way can you use?). From this form (product-of-sums I mean) you can obtain the corresponding OR-AND circuit; (see example on pages 2, 3 and figures 3, 4 of handout 7).

• <u>Transforming an OR-AND logic circuit into an AND-OR logic circuit.</u>

<u>Do the following:</u> Draw the figure of the OR-AND logic circuit. Compute its output F. F is going to be in product-of-sums form. Transform F into sum-of-products form by multiplying out; (what other algebraic way can you use to do this?). From this form (sum-of-products I mean) you can obtain the corresponding AND-OR logic circuit; (see example on pages 1, 2 and figures 1, 2 of handout# 7).

<u>Note:</u> The above transformations rely on factoring and multiplying out, for example. Sometimes, factoring and multiplying out can be difficult. There

is another algebraic way of performing the above two transformations; (which one is it?). This algebraic way, however, could be time consuming. The above two transformations can also be performed (I mean transforming an AND-OR circuit into an OR-AND circuit and vice versa) using the graphical approach (bubbles etc...) instead of algebraic approaches. Two examples follow demonstrating this.

Example 1: Consider the logic circuit of figure 1 below. It is an AND-OR logic circuit. Transform the circuit of fig. 1 into the corresponding OR-AND logic circuit by using the graphical approach.

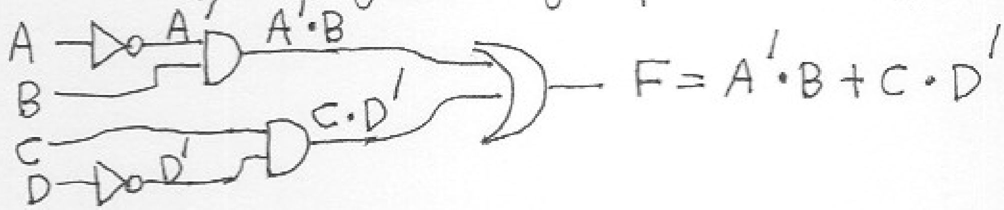


Figure 1; (An AND-OR logic circuit)

Answer: The corresponding OR-AND logic circuit is shown by figure 2 below.

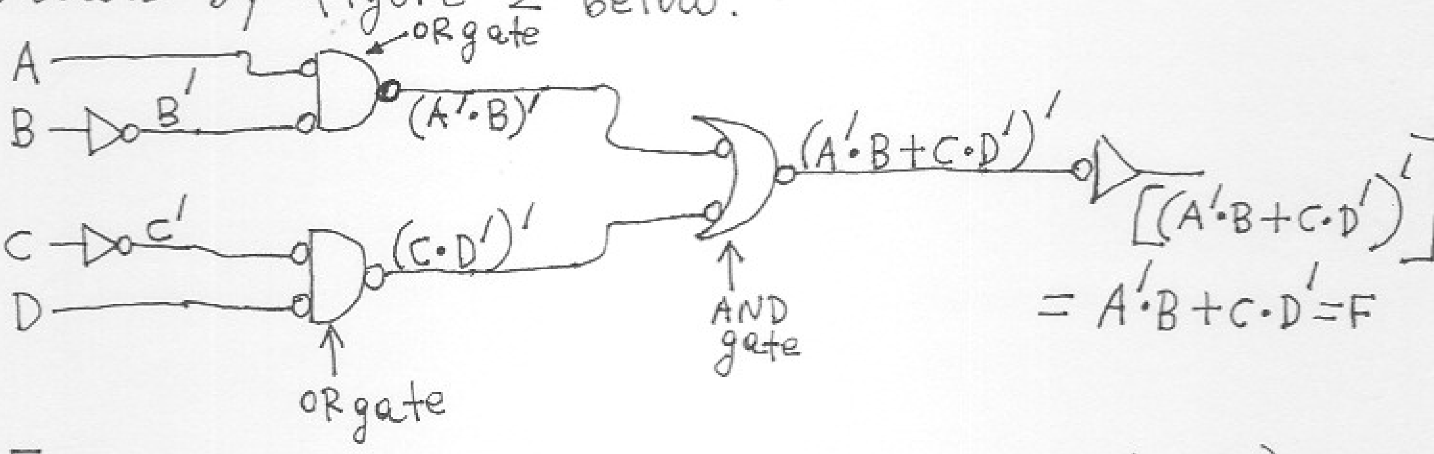


Figure 2; (the corresponding OR-AND logic circuit).

Example 2: Consider the logic circuit of figure 3 below. It is an OR-AND logic circuit. Transform the circuit of fig. 3 into the corresponding AND-OR logic circuit by using the graphical approach.

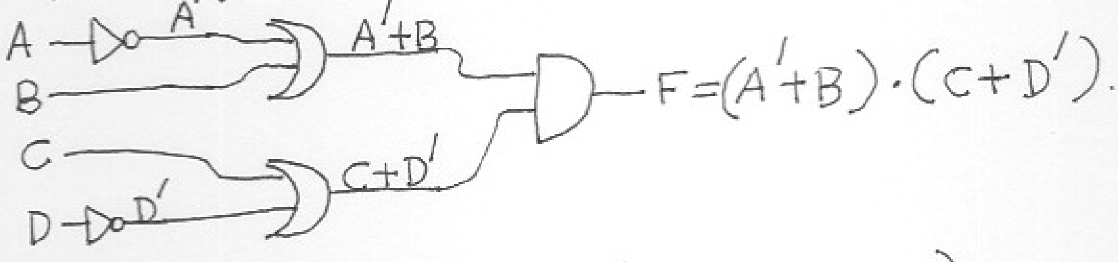


Figure 3; (An OR-AND logic circuit)

Answer: The corresponding AND-OR logic circuit is shown by figure 4 below.

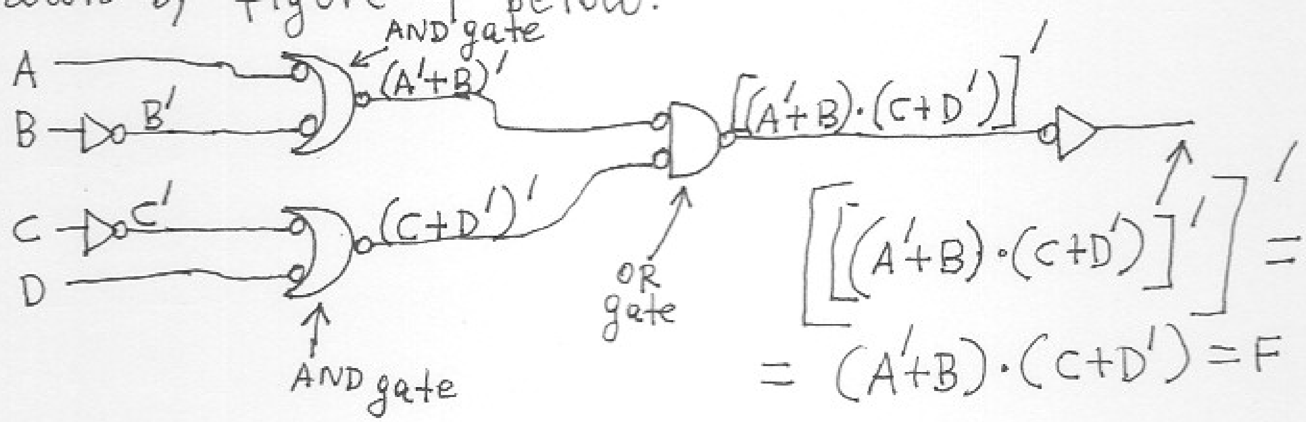


Figure 4; (the corresponding AND-OR logic circuit).

Note: By now you should have become experts with the graphical approach.

• The 2-input Exclusive-OR (XOR) gate and the 2-input Exclusive NOR (XNOR) gate

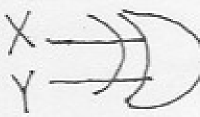
A 2-input Exclusive-OR (XOR) gate is a 2-input gate whose output is 1 if exactly one of its inputs is 1. In other words, a 2-input XOR gate produces a 1 output if its inputs are different. The XOR operation is denoted by the symbol "$\oplus$".

A 2-input Exclusive NOR (XNOR) or Equivalence gate is a 2-input gate whose output is the complement of the output of the 2-input XOR gate that has the same inputs. In other words, a 2-input XNOR gate produces a 1 output if its inputs are the same.

The truth tables, logic equations and symbols (official and equivalent symbols) for the XOR and XNOR gate follow:

X	Y	$X \oplus Y$ (XOR)
0	0	0
0	1	1
1	0	1
1	1	0

Table 1; (truth table of a 2-input XOR gate).



$Z = X \oplus Y$

Figure 5; ((official) symbol for a 2-input XOR gate).

I now provide the logic equations for the 2-input XOR gate; (I am providing these detailed explanations for the last time. We did it before in handout #8).

As seen from table 1 above, $X \oplus Y$ is 1 if ($X=0$ and $Y=1$) or ($X=1$ and $Y=0$). In other words $X \oplus Y$ is 1 if ($X'=1$ and $Y=1$) or ($X=1$ and $Y'=1$).

This can be written as a logic equation as shown below

$$X \oplus Y = X' \cdot Y + X \cdot Y' = \Sigma_{X,Y}(1,2) = \Pi_{X,Y}(0,3) = (X+Y)\cdot(X'+Y')$$

So I have expressed $X \oplus Y$ in two alternative forms. They are:

$$X \oplus Y = X' \cdot Y + X \cdot Y' \quad (1)$$

$$X \oplus Y = (X+Y) \cdot (X'+Y') \quad (2).$$

Three equivalent symbols for a 2-input XOR gate follow. They are shown in figures 6, 7, 8 below.

Figure 6; (equivalent symbol # 1 for a 2-input XOR gate)

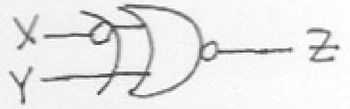


Figure 7; (equivalent symbol # 2 for a 2-input XOR gate)

Figure 8; (equivalent symbol # 3 for a 2-input XOR gate).

Let me now prove that the outputs produced by each of the gates of figures 6, 7, 8 above is $X \oplus Y$. This way I would have proved that the symbols of figures 6, 7, 8 are equivalent to the symbol of fig. 5; (they all represent XOR gate).

Consider figure 6. We have

$$Z = X' \oplus Y' = (X')' \cdot Y' + X' \cdot (Y')' \quad \text{(according to (1) above)}$$

$$= X \cdot Y' + X' \cdot Y = X' \cdot Y + X \cdot Y' = X \oplus Y \quad \text{(see eq. (1) above).}$$

Consider figure 7. We have

$$Z = (X' \oplus Y)' = [(X')' \cdot Y + X' \cdot Y']' \quad \text{(according to (1) above)}$$

$$= (X \cdot Y + X' \cdot Y')' = (X'+Y') \cdot (X+Y) = (X+Y) \cdot (X'+Y') =$$

$$= X \oplus Y \quad \text{(see eq. (2) above).}$$

Consider now figure 8. We have.

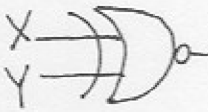
$Z=(X\oplus Y')' = [X'\cdot Y' + X\cdot (Y')']'$ (according to (1) of previous page)

$=(X'\cdot Y' + X\cdot Y)' = (X+Y)\cdot(X'+Y') = X\oplus Y$ (see eq. (2) of previous page).

I now provide the truth table, logic equations and symbols (official and equivalent symbols) for a 2-input XNOR gate.

X Y	$(X\oplus Y)'$ (XNOR)
0 0	1
0 1	0
1 0	0
1 1	1

Table 2; (truth table of a 2-input XNOR gate).



$Z=(X\oplus Y)'$

Figure 9; ((official) symbol for a 2-input XNOR gate).

The logic equations for the 2-input XNOR gate are now provided. From the above truth table 2 one can easily get:

$(X\oplus Y)' = X'\cdot Y' + X\cdot Y = \Sigma_{X,Y}(0,3) = \Pi_{X,Y}(1,2) = (X+Y')\cdot(X'+Y)$.

So $(X\oplus Y)'$ has been expressed in two alternative forms. They are:

$(X\oplus Y)' = X'\cdot Y' + X\cdot Y$ (3)

$(X\oplus Y)' = (X+Y')\cdot(X'+Y)$ (4).

Three equivalent symbols for a 2-input XNOR gate follow. They are shown in figures 10, 11, 12 on next page.

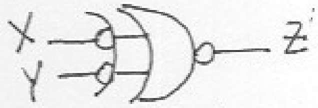


Figure 10; (equivalent symbol # 1 for a 2-input XNOR gate)

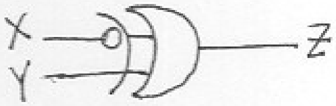


Figure 11; (equivalent symbol #2 for a 2-input XNOR gate).

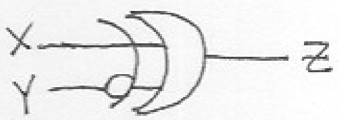


Figure 12; (equivalent symbol #3 for a 2-input XNOR gate).

Let me now prove that the output produced by each of the gates of figures 10, 11, 12 is $(X \oplus Y)'$. This way I would have proved that the symbols of figures 10, 11, 12 are equivalent to the symbol of figure 9; (they all represent XNOR gate).

Consider figure 10. We have.

$Z = (X' \oplus Y')' = (X')' \cdot (Y')' + X' \cdot Y'$ (according to (3) of previous page)

$= X' \cdot Y' + X \cdot Y = (X \oplus Y)'$ (see eq. (3) of previous page).

Consider figure 11. We have

$Z = X' \oplus Y = (X')' \cdot Y + X' \cdot Y'$ (according to (1) on page 5)

$= X \cdot Y + X' \cdot Y' = X' \cdot Y' + X \cdot Y = (X \oplus Y)'$ (see (3) of page 6).

Consider now figure 12. We have.

$Z = X \oplus Y' = X' \cdot Y' + X \cdot (Y')'$ (according to (1) on page 5)

$= X' \cdot Y' + X \cdot Y = (X \oplus Y)'$ (see (3) on page 6).

The following theorems apply to Exclusive-OR (XOR):

$X \oplus 0 = X$, (5)

$X \oplus 1 = X'$ (6)

$X \oplus X = 0$ (7)

$X \oplus X' = 1$ (8)

$X \oplus Y = Y \oplus X$ (9) commutativity

$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$ (10) associativity

$X \cdot (Y \oplus Z) = X \cdot Y \oplus X \cdot Z$ (11) distributivity

$(X \oplus Y)' = X \oplus Y' = X' \oplus Y$ (12).

Note: We have already proved the above equation (12). See page 7.

Problem: Prove eqs. (5), (6), (7), (8), (9), (10), (11). I will probably give you these as a homework problem.

Note: Commutativity and associativity also apply to the XNOR operation.

Note: In order to simplify an expression that contains AND, OR, XOR and XNOR operations it is usually desirable to first apply equation (1) and equation (3) to eliminate the XOR and XNOR operations. An example follows:

Example 3: Simplify $F = (A' \cdot B \oplus C)' + (B \oplus A \cdot C')$

Answer: By eqs (1) and (3) we have

$$F = (A' \cdot B \oplus C)' + (B \oplus A \cdot C') = [(A' \cdot B)' \cdot C' + (A' \cdot B) \cdot C] +$$

$$+ [B' \cdot (A \cdot C') + B \cdot (A \cdot C')'] = (A + B') \cdot C' + A' \cdot B \cdot C +$$

$$+ A \cdot B' \cdot C' + B \cdot (A' + C) = B \cdot (A' \cdot C + A' + C) + C' \cdot (A + B' + A \cdot B')$$

$$= B \cdot [A' \cdot C + A' \cdot 1 + C] + C' \cdot [A + B' \cdot 1 + B' \cdot A] =$$

$$= B \cdot [A' \cdot (C + 1) + C] + C' \cdot [A + B' (1 + A)] =$$

$$= B \cdot [A' \cdot 1 + C] + C' \cdot [A + B' \cdot 1] = B \cdot (A' + C) + C' \cdot (A + B').$$

Note:

- When manipulating an expression that contains several XOR or XNOR operations, it is useful to consider the equation below

$$(X'\cdot Y + X\cdot Y')' = X'\cdot Y' + X\cdot Y \qquad (13)$$

We don't need to prove eq. (13). Both left and right side of (13) represent the XNOR operation. Just see eqs. (1) and (3).

Example 4: Simplify $F = A' \oplus B \oplus C$

Answer: We have $F = A' \oplus B \oplus C = (A' \oplus B) \oplus C =$

$= ((A')'\cdot B + A'\cdot B') \oplus C$ (according to (1))

$= (A\cdot B + A'\cdot B') \oplus C = (A\cdot B + A'\cdot B')'\cdot C + (A\cdot B + A'\cdot B')\cdot C'$ (according to (1))

$= (A'\cdot B + A\cdot B')\cdot C + (A\cdot B + A'\cdot B')\cdot C'$ (according to (13))

$= A'\cdot B\cdot C + A\cdot B'\cdot C + A\cdot B\cdot C' + A'\cdot B'\cdot C'$