

EE 2720

Handout # 10

Remember: The two equivalent logic symbols for an n -input NAND gate are the ones shown by Figures 1 and 2 below.

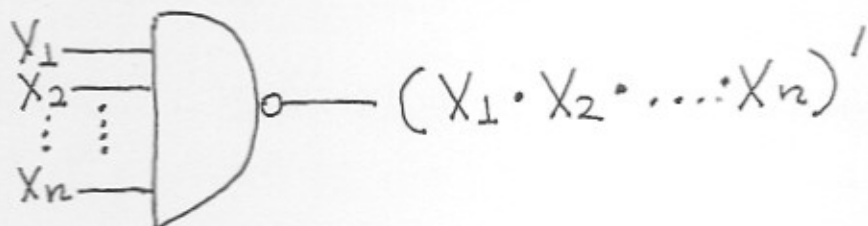


Figure 1; (logic symbol for an n -input NAND gate).

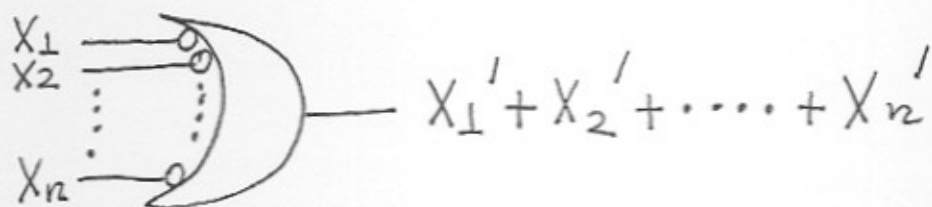


Figure 2; (equivalent logic symbol for an n -input NAND gate)

Remember: The two equivalent logic symbols for an n -input NOR gate are the ones shown by Figures 3 and 4 below.

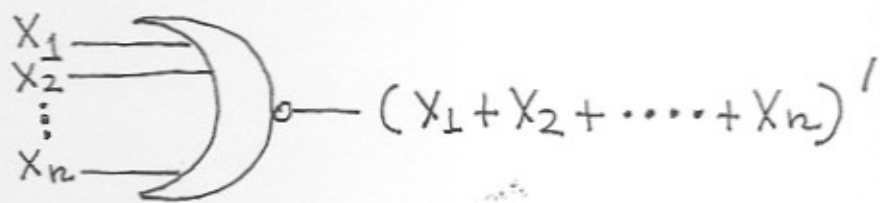


Figure 3; (logic symbol for an n -input NOR gate)

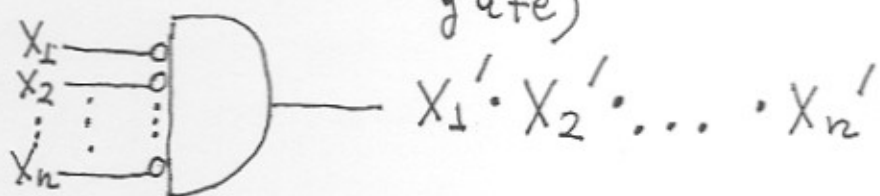


Figure 4; (equivalent logic symbol for an n -input NOR gate).

In handout # 9, we stated and proved that ② any logic function can be realized using only NAND gates or only NOR gates. An obvious way of realizing any logic function using only NAND or only NOR gates is by replacing each AND or OR or NOT gate by a NAND or a NOR gate; (you have to draw a figure of the function first). This method results in redundant gates which would then should be eliminated. To replace the AND, OR, NOT gates by NAND or NOR gates, see figures 11, 12, 13, 14, 15, 16 on pages 7, 8, 9 of handout # 9.

A more elegant way of performing the above task follows:

• Realization of a logic function using only NAND gates

In order to realize a logic function using only NAND gates do the following: Let the function be F . Translate the function F in sum-of-products form; (you can do this, for example, by multiplying out). Write the function F as $F = (F')'$. Apply DeMorgan's theorem once; (only once). The following example clarifies the above.

Example 1: Realize the function $F = A \cdot C + A' \cdot B \cdot D' + A' \cdot B \cdot E + A' \cdot C' \cdot D \cdot E$ using only NAND gates

Answer: The given function F is already in sum-of-products form. F can be written as:

$$F = \left[(A \cdot C + A' \cdot B \cdot D' + A' \cdot B \cdot E + A' \cdot C' \cdot D \cdot E)' \right]' = \quad (3)$$

$$= \left[(A \cdot C)' \cdot (A' \cdot B \cdot D')' \cdot (A' \cdot B \cdot E)' \cdot (A' \cdot C' \cdot D \cdot E)' \right]'$$

We now provide a figure showing the realization of F using only NAND gates.

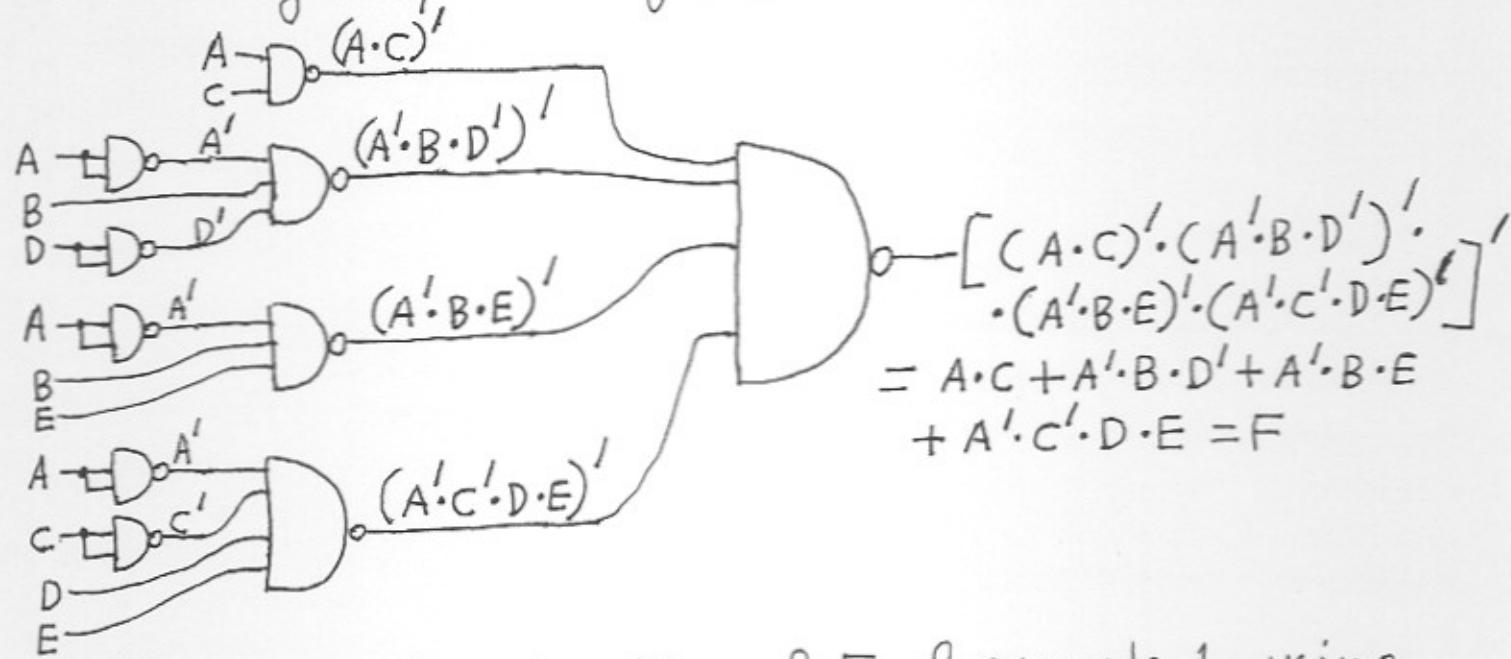


Figure 5; (realization of F of example 1 using only NAND gates; the NAND gates are arranged in 3 levels and there are 10 NAND gates; wrong count why?).

Lets now repeat the same example starting from the original expression of F by replacing the AND, OR, NOT gates by NAND gates by using figures 11, 12, 13 on pages 7, 8 of handout # 9. We will first provide a figure showing the original implementation of the function F. As a reminder, this function is $F = A \cdot C + A' \cdot B \cdot D' + A' \cdot B \cdot E + A' \cdot C' \cdot D \cdot E$. The figure is shown on the next page.

(4)

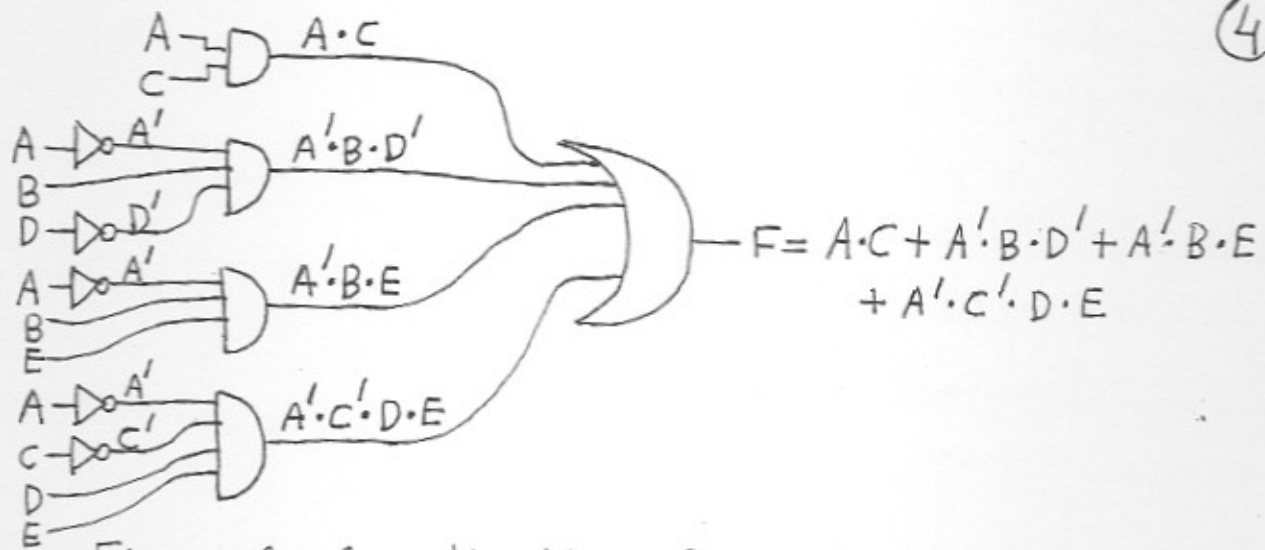


Figure 6 ; (realization of F using AND, OR, NOT gates).

We now translate figure 6 into a figure consisting of only NAND gates by using figures 11, 12, 13 on pages 7, 8 of handout # 9.

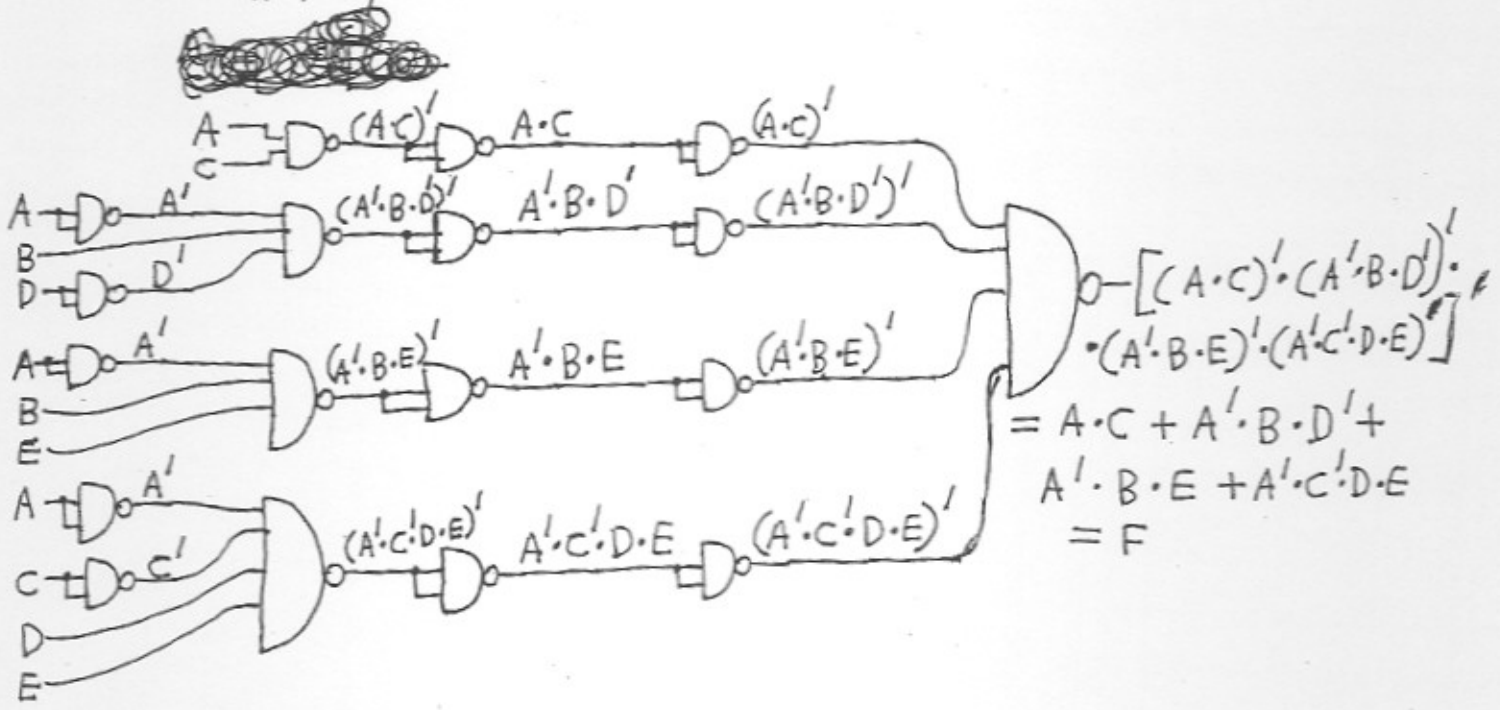


Figure 7; (another realization of F using only NAND gates; here the NAND gates are arranged in 5 levels and there are 18 NAND gates; wrong counts why?).

⑤ Taking a close look at figure 7, one can see that 8 NAND gates can be eliminated and replaced by wires; (which are they?). Then figure 7 becomes identical to figure 5. That is why it is not a good idea to replace AND, OR, NOT gates by NAND gates using figures 11, 12, 13 of handout # 9. You do redundant work. You produce redundant gates which must then be eliminated. This concludes example 1.

Question: Do you want me to make your life easier?

Answer: of course yes.

Then look at the example below.

Example 2: Realize the function $F = A \cdot C + A' \cdot B \cdot D' + A' \cdot B \cdot E + A' \cdot C' \cdot D \cdot E$ using only NAND gates.

Answer: The given function F is already in sum-of-products form. It is the same function as this of example 1. Two figures follow below.

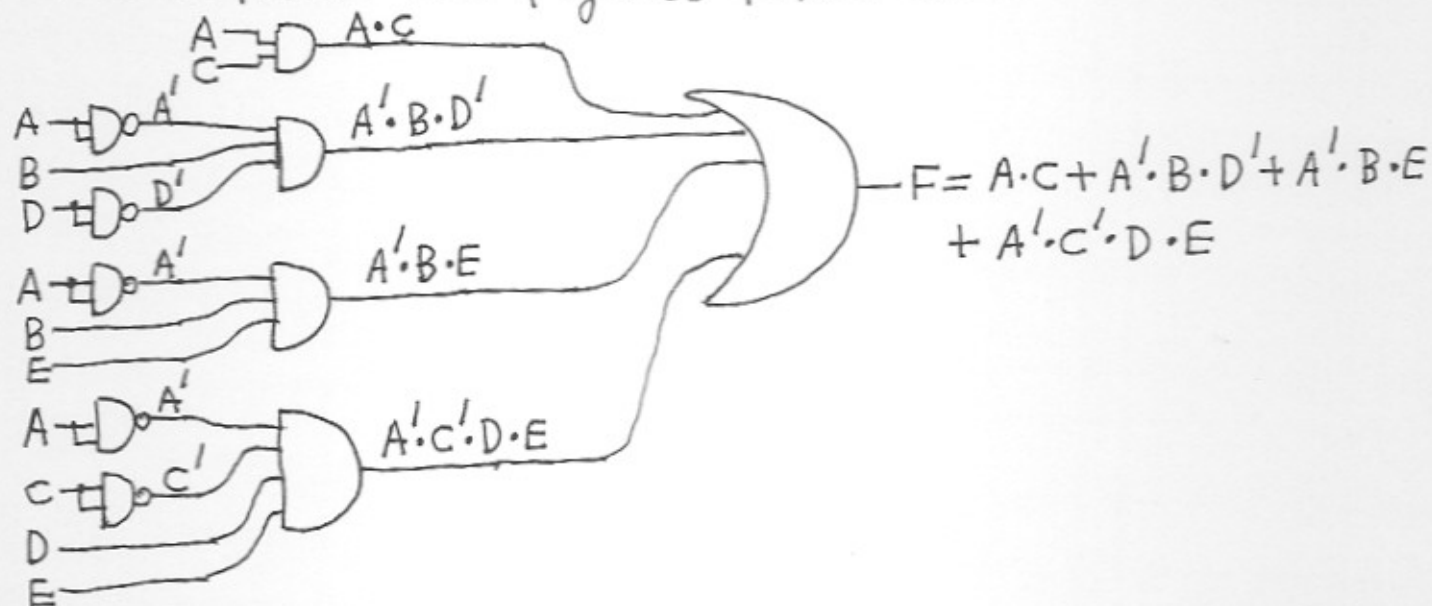


Figure 8; (realization of F ; here the NOT gates have been replaced by NAND gates but the AND and OR gates remain).

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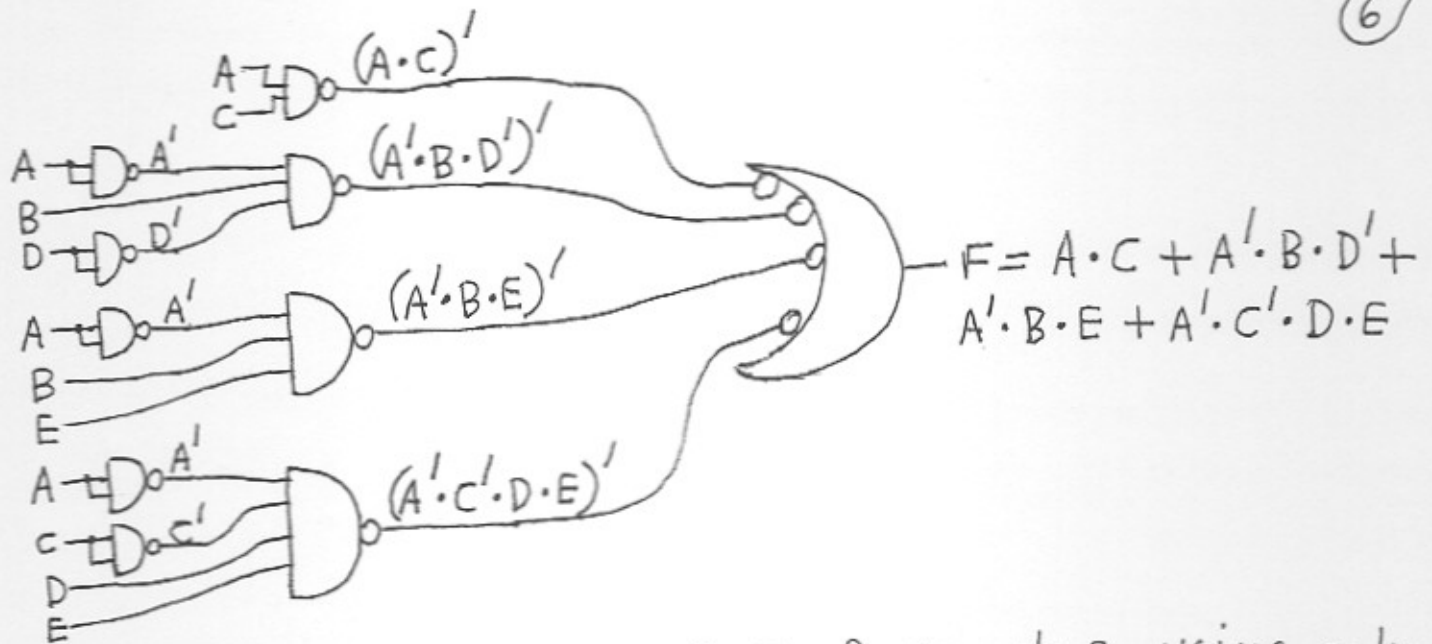


Figure 9; (realization of F of example 2 using only NAND gates).

Note: In this example I used a graphical approach instead of algebraic. I just inserted inversion bubbles. Again, for this approach ~~the function must be transformed into sum-of-products form~~, the function must be transformed into sum-of-products form; (I mean the approach of using only NAND gates; the graphical approach).

• Realization of a logic function using only NOR gates

In order to realize a logic function using only NOR gates do the following: Let the function be F . Translate the function F in product-of-sums form; (you can do this, for example, by factoring). Write the function F as $F = (F')'$. Apply DeMorgan's theorem only once.

The following example clarifies the above.

Example 3: Realize the function

$F = (A+B+C') \cdot (A+B+D) \cdot (A+B+E) \cdot (A+D'+E) \cdot (A'+C)$ using only NOR gates.

Answer: The given function F is already in product-of-sums form. F can be written as: ⑦

$$F = (A+B+C') \cdot (A+B+D) \cdot (A+B+E) \cdot (A+D'+E) \cdot (A'+C) =$$

$$= \left[\left[(A+B+C') \cdot (A+B+D) \cdot (A+B+E) \cdot (A+D'+E) \cdot (A'+C) \right]' \right]' =$$

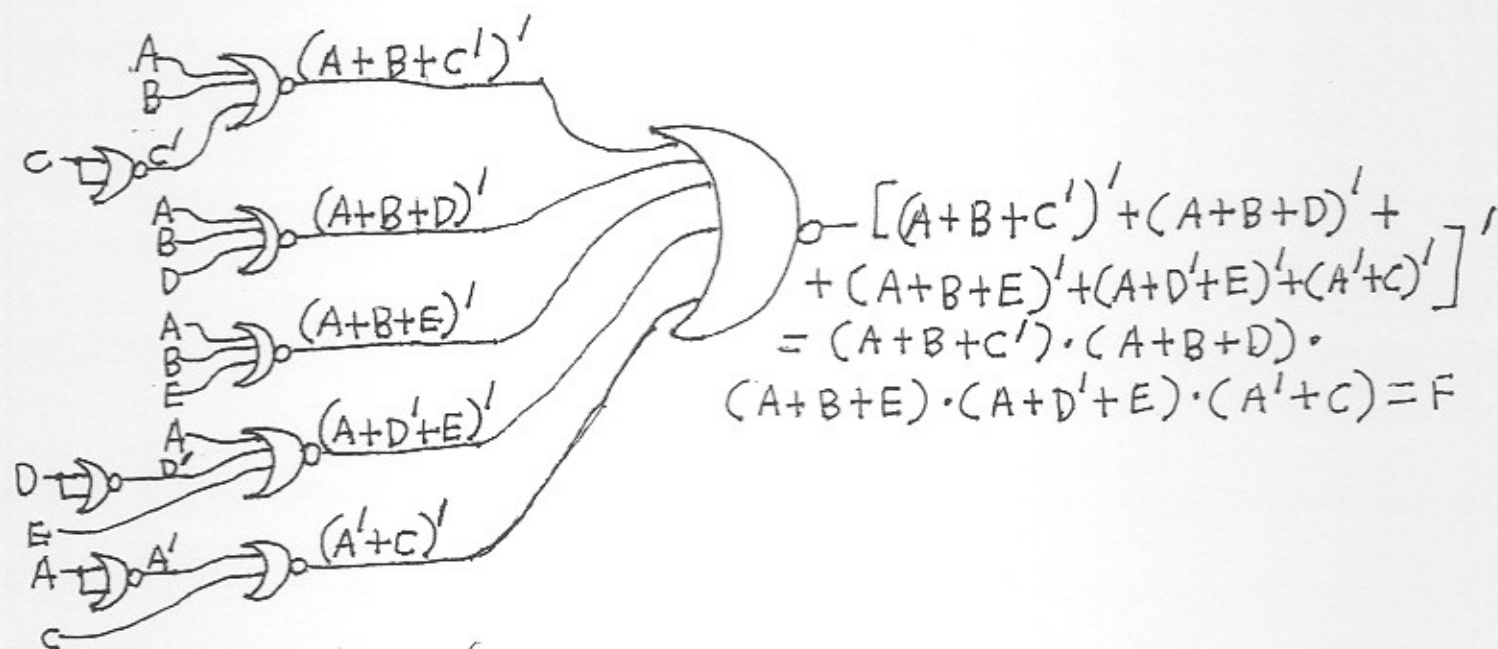
$$= \left[(A+B+C')' + (A+B+D)' + (A+B+E)' + (A+D'+E)' + (A'+C)' \right]'$$


Figure 10; (realization of F of example 3 using only NOR gates; the NOR gates are arranged in 3 levels and there are 9 NOR gates).

Note: If you were to repeat the same example by replacing the AND, OR, NOT gates by NOR gates by using figures 14, 15, 16 on pages 8, 9 of handout # 9, you would end up with a figure consisting of 19 NOR gates arranged in 5 levels; (do this if you want as a homework problem). 10 of these NOR gates can be eliminated (they are redundant) and replaced by wires. Then you will get again the above figure 10. That is why it is not a good idea to replace AND, OR, NOT gates by NOR gates using figures 14, 15, 16 of handout # 9. You do redundant

work. You produce redundant gates which must ⁸ then be eliminated.

I will now repeat the same example 3 using the graphical approach. As a reminder, the function that we want to realize using only NOR gates is $F = (A+B+C') \cdot (A+B+D) \cdot (A+B+E) \cdot (A+D'+E) \cdot (A'+C)$. Two figures follow.

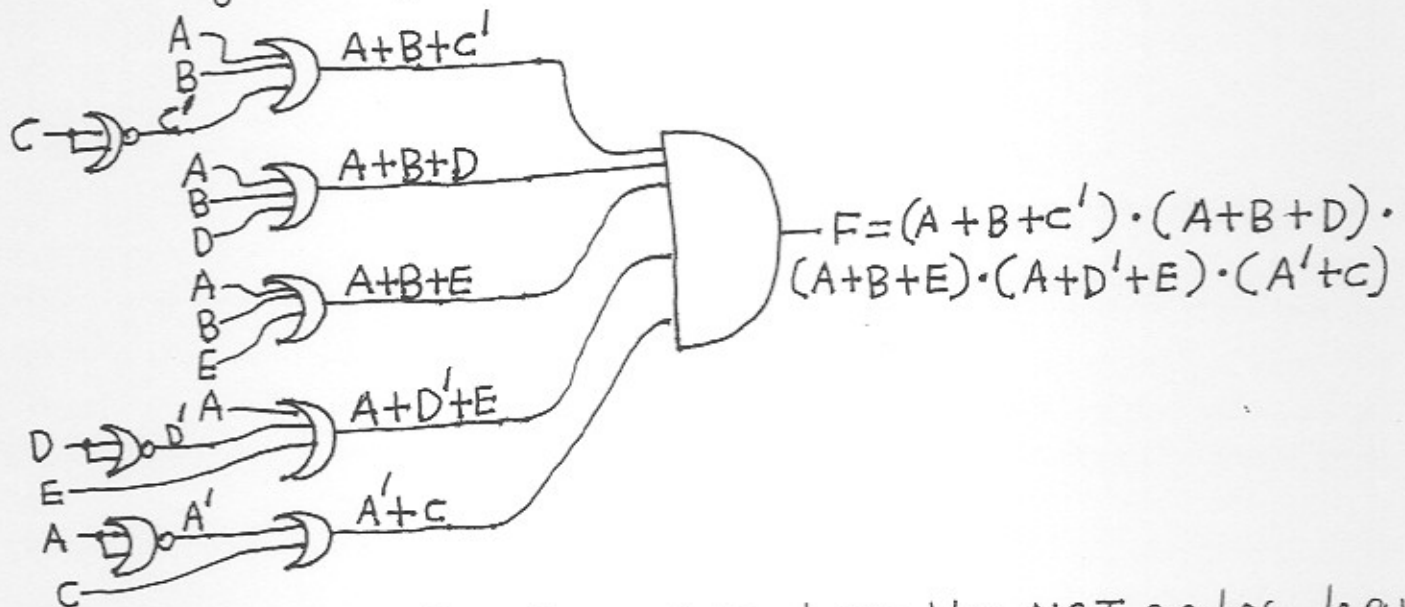


Figure 11; (realization of F ; here the NOT gates have been replaced by NOR gates but the AND and OR gates remain). Now I insert bubbles to get the figure below.

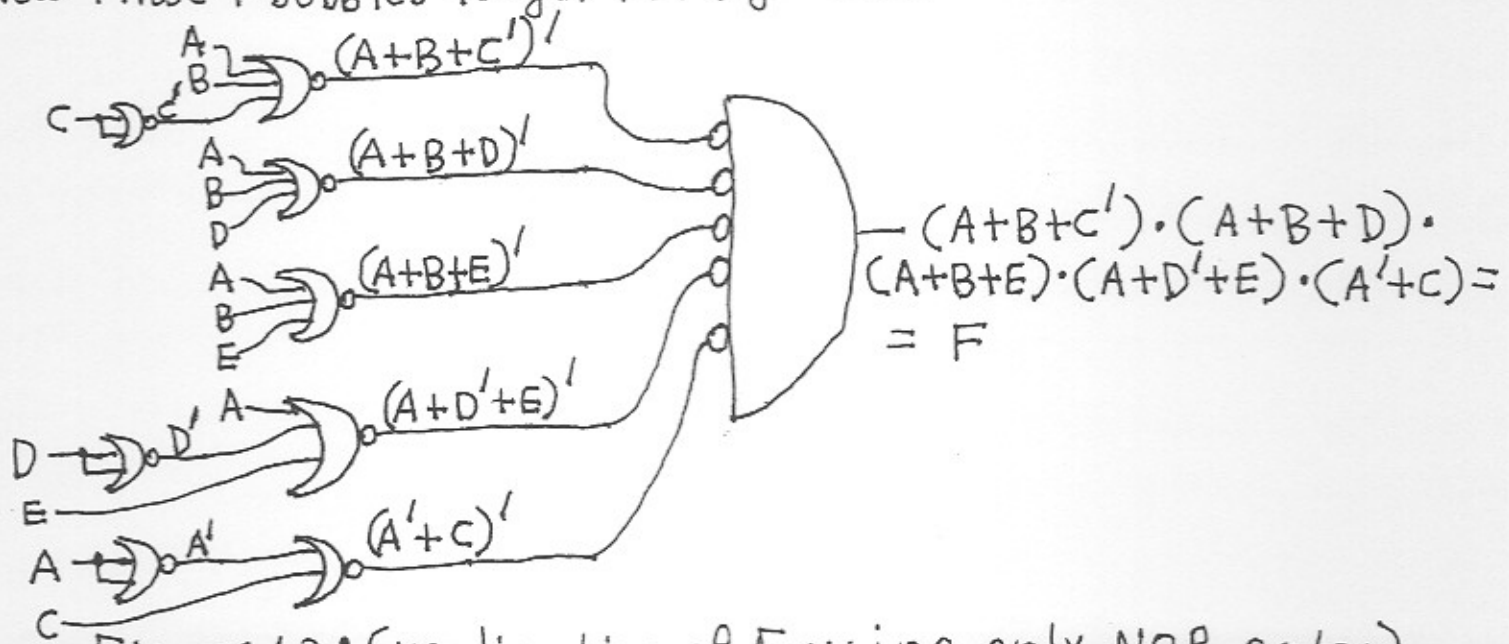


Figure 12; (realization of F using only NOR gates).

Note: When using the graphical approach in (9) order to realize a function using only NOR gates, the function must be transformed into product-of-sums form.

Example 4: Realize the function provided below using only NAND gates. Then realize it using only NOR gates. The function is:

$$F = A + B \cdot C' + B' \cdot C \cdot D$$

Answer: I will first realize F using only NAND gates. Then I will realize it using only NOR gates. I will follow the graphical approach for both cases. The given function F is in sum-of-products form, so we are ready to realize it using only NAND gates. Two figures follow

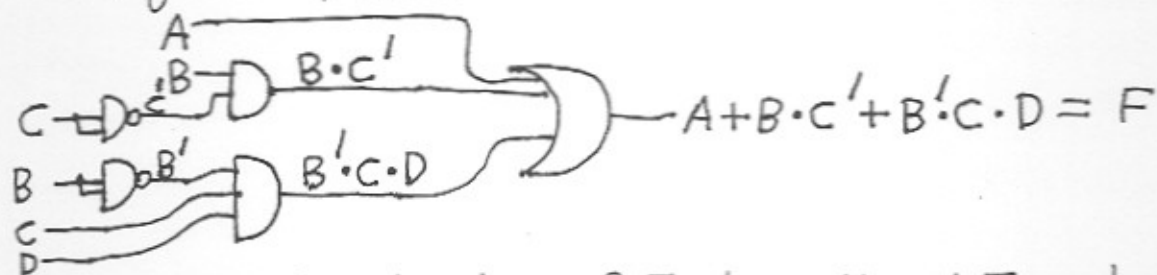


Figure 13; (realization of F ; here the NOT gates have been replaced by NAND gates but the AND and OR gates remain)

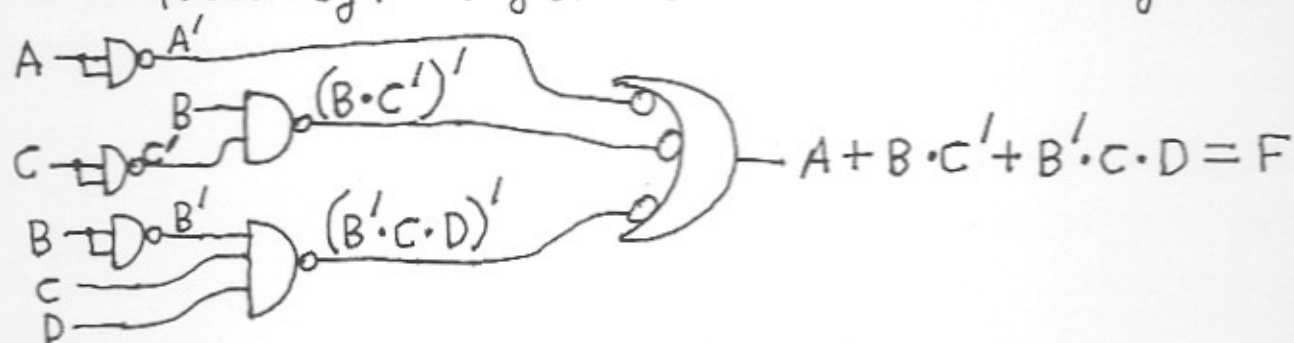


Figure 14; (realization of F using only NAND gates)

For the above realization of F of figure 14, we had 6 NAND gates arranged in 3 levels. We now realize the function F using only NOR gates. We must first transform F into product-

of-sums form. The way I will do this is by first ⁽¹⁰⁾ writing the function F in canonical sum form (sum of minterms). From the canonical sum form I will easily find the canonical product form (product of maxterms) which will give me a product-of-sums form for F. We have:

$$\begin{aligned}
 F &= A + B \cdot C' + B' \cdot C \cdot D = A \cdot \underbrace{(B+B')} \cdot \underbrace{(C+C')} \cdot \underbrace{(D+D')} + \\
 &+ B \cdot C' \cdot \underbrace{(A+A')} \cdot \underbrace{(D+D')} + B' \cdot C \cdot D \cdot \underbrace{(A+A')} = \\
 &= (A \cdot B + A \cdot B') \cdot (C \cdot D + C \cdot D' + C' \cdot D + C' \cdot D') + B \cdot C' \cdot A \cdot D + \\
 &+ B \cdot C' \cdot A \cdot D' + B \cdot C' \cdot A' \cdot D + B \cdot C' \cdot A' \cdot D' + B' \cdot C \cdot D \cdot A + B' \cdot C \cdot D \cdot A' \\
 &= \underbrace{A \cdot B \cdot C \cdot D} + \underbrace{A \cdot B \cdot C \cdot D'} + \underbrace{A \cdot B \cdot C' \cdot D} + \underbrace{A \cdot B \cdot C' \cdot D'} + \underbrace{A \cdot B' \cdot C \cdot D} + \\
 &+ \underbrace{A \cdot B' \cdot C \cdot D'} + \underbrace{A \cdot B' \cdot C' \cdot D} + \underbrace{A \cdot B' \cdot C' \cdot D'} + \underbrace{A \cdot B \cdot C' \cdot D} + \underbrace{A \cdot B \cdot C' \cdot D'} + \\
 &+ \underbrace{A' \cdot B \cdot C \cdot D} + \underbrace{A' \cdot B \cdot C' \cdot D'} + \underbrace{A \cdot B' \cdot C \cdot D} + \underbrace{A' \cdot B' \cdot C \cdot D}
 \end{aligned}$$

$$= \sum_{A,B,C,D} (3, 4, 5, 8, 9, 10, 11, 12, 13, 14, 15) =$$

$$\prod_{A,B,C,D} (0, 1, 2, 6, 7) =$$

$$= (A+B+C+D) \cdot (A+B+C+D') \cdot (A+B+C'+D) \cdot (A+B'+C'+D) \cdot (A+B'+C'+D')$$

We now have the function F in product-of-sums form. This is

$$F = (A+B+C+D) \cdot (A+B+C+D') \cdot (A+B+C'+D) \cdot (A+B'+C'+D) \cdot (A+B'+C'+D')$$

We are now ready to realize F using only NOR gates.

Two figures follow

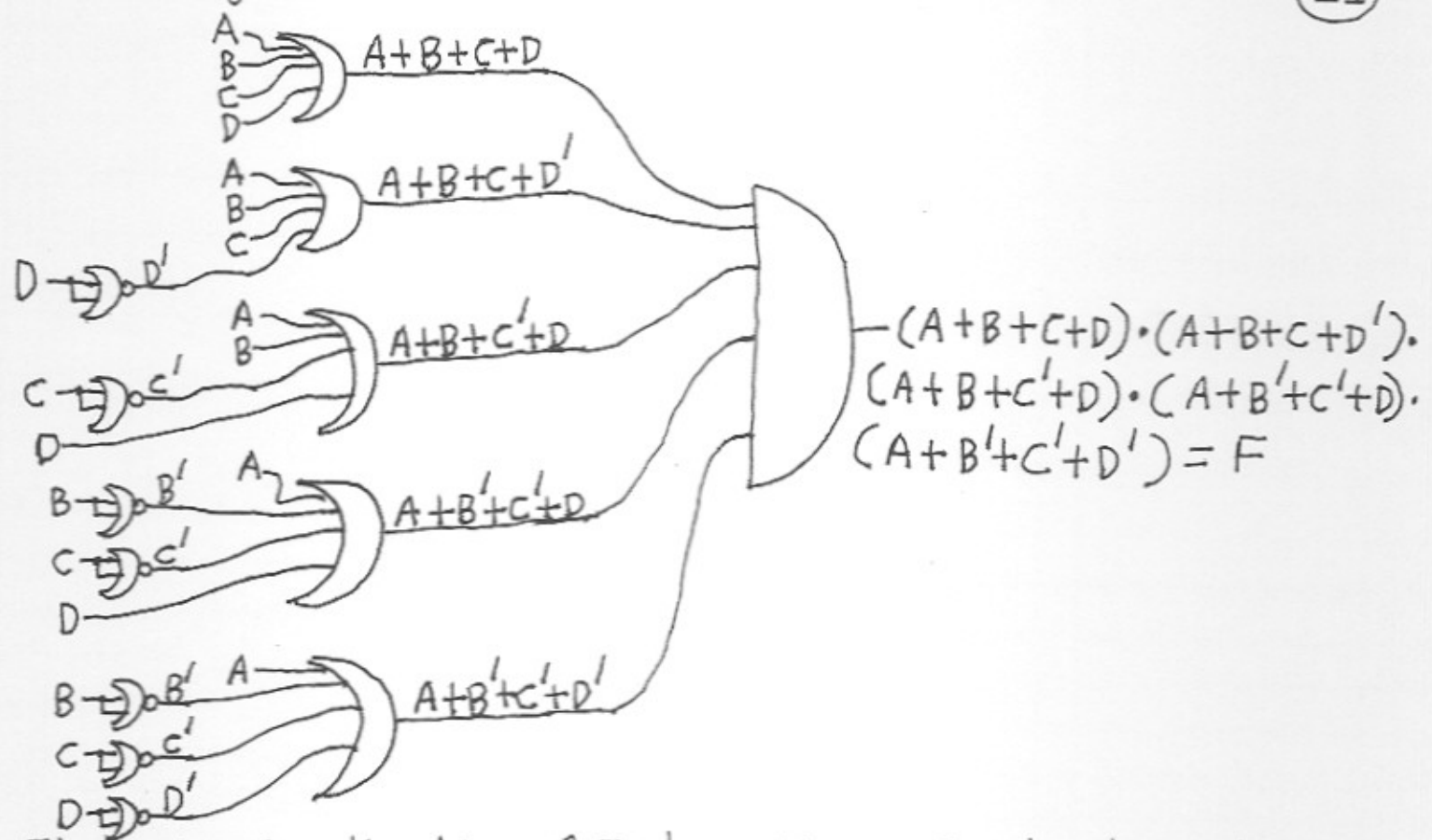


Figure 15; (realization of F ; here the NOT gates have been replaced by NOR gates but the AND and OR gates remain).

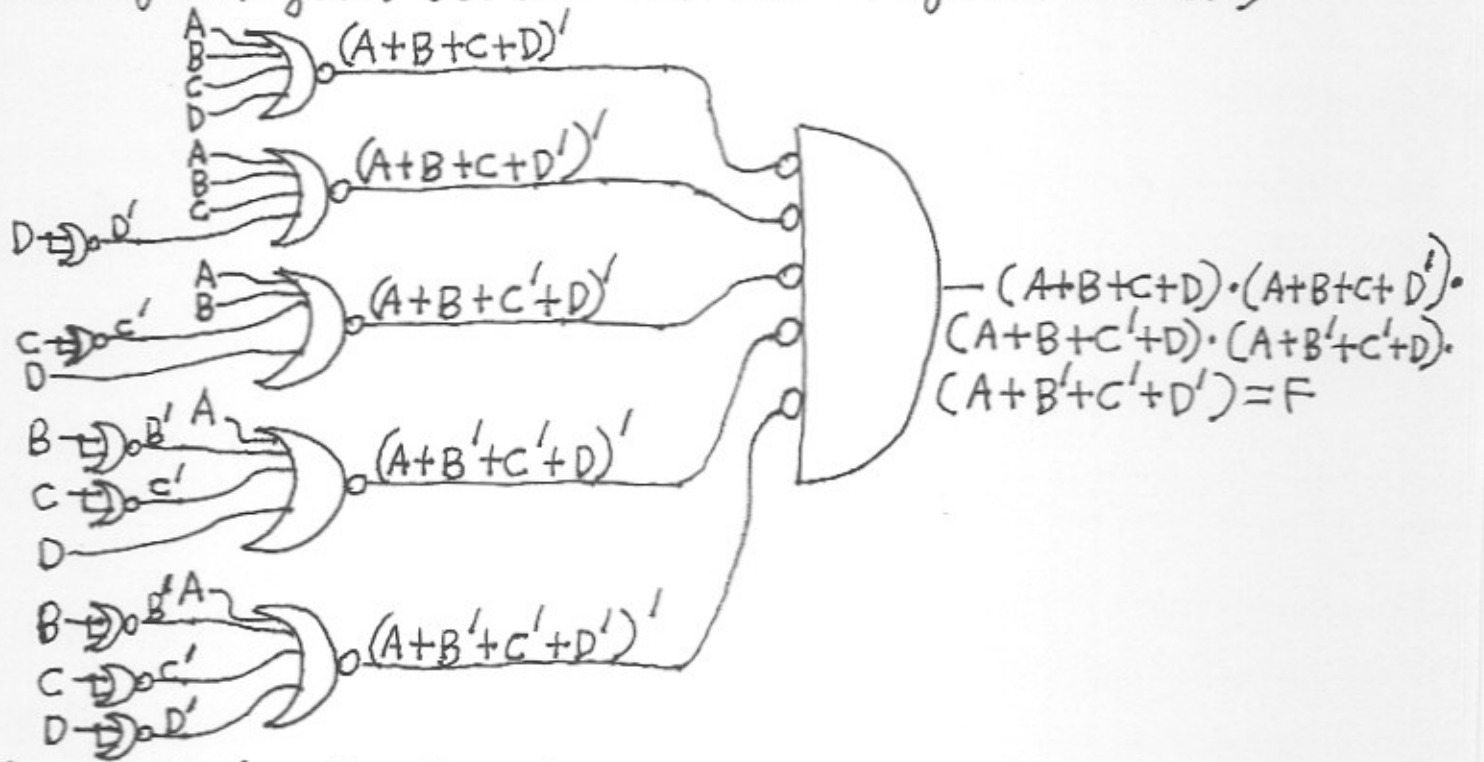


Figure 16; (realization of F using only NOR gates; 3 levels of NOR gates).

Note: When we realize a logic function F using only (12) NAND gates or only NOR gates, we need 2 or 3 levels of NAND gates or NOR gates; (no less than 2 levels and no more than 3 levels). We need 3 levels, if the function F contains complemented variables because in this case we need inverters (NOT gates) which are realized either by NAND or NOR gates. Otherwise, if the function F doesn't contain complemented variables, we only need 2 levels of NAND or NOR gates.

Note: In some of the previous examples of realizing different logic functions using only NAND gates or only NOR gates, I provided the wrong count of NAND and NOR gates. Can you tell why?

• More Information

Note: Any logic function F expressed in sum-of-products form can be realized in the following four forms:

1. It can be realized using NOT, AND and OR gates; (you already know that any logic function can be realized using NOT, AND and OR gates).
2. It can be realized using only NAND gates (you already know that any logic function can be realized using only NAND gates).
3. It can be realized using only OR and NAND gates; (since any logic function can be transformed into sum-of-products form, that means that any logic function can be realized using only OR and NAND gates. It is only that you must transform the function into sum-of-products first before you realize it using only OR and NAND gates).
4. It can be realized using only NOR and OR gates; (since any logic function can be transformed into sum-of-products form, that means that any logic function can be realized using only NOR and OR gates. It is only that

you must transform the function into sum-of-products (13) first before you realize it using only NOR and OR gates.

Note: Any logic function F expressed in product-of-sums form can be realized in the following four forms provided below. Since any logic function can be transformed into product-of-sums form, that means that any logic function can be realized in either one of the four forms provided below. It is only that you must transform the function into product-of-sums form first before you realize it in either one of the four forms provided below. The forms are:

1. It can be realized using NOT, OR and AND gates; (you already know this).
2. It can be realized using only NOR gates; (you already know this)
3. It can be realized using only AND and NOR gates.
4. It can be realized using only NAND and AND gates.

I will demonstrate the above with two examples.

Example 5: Consider the logic function F where $F = A + B \cdot C' + B' \cdot C \cdot D$. Realize the function F

1. Using NOT, AND and OR gates; (this is going to be realization #1).
2. Using only NAND gates; (this is going to be realization #2)
3. Using only OR and NAND gates; (this is going to be realization #3).
4. Using only NOR and OR gates; (this is going to be realization #4).

Answer: The given logic function F is already in sum-of-products form. We have:

$$F = A + B \cdot C' + B' \cdot C \cdot D \quad (1)$$

$$= \left[(A + B \cdot C' + B' \cdot C \cdot D)' \right]' =$$

$$= \left[A' \cdot (B \cdot C')' \cdot (B' \cdot C \cdot D)' \right]' \quad (2)$$

$$= \left[A' \cdot (B' + C) \cdot (B + C' + D') \right]' \quad (3)$$

$$= A + (B' + C)' + (B + C' + D')' \quad (4)$$

Equation (1) above will give us realization # 1, equation (2) will give us realization # 2, equation (3) will give us realization # 3 and equation (4) will give us realization # 4. The respective figures showing these four realizations follow:

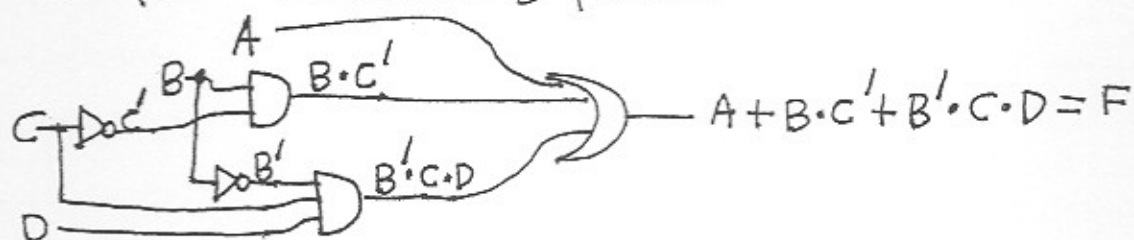


Figure 17; (realization of F using NOT, AND and OR gates. Here equation (1) above was used).

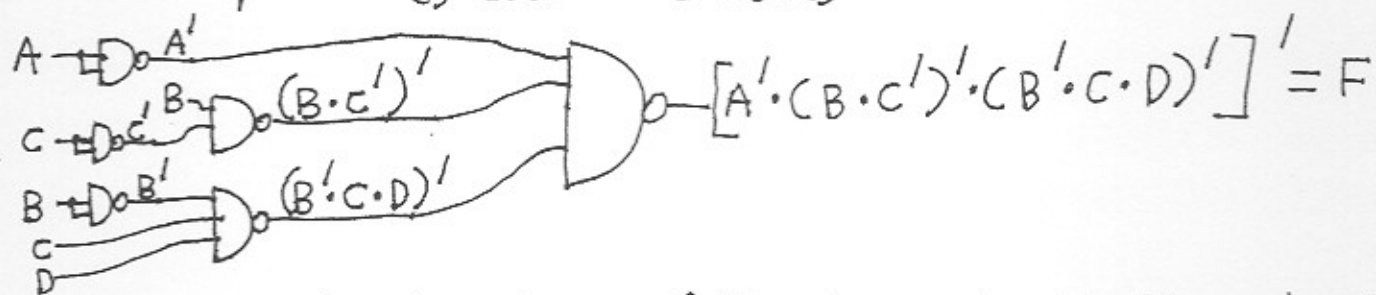


Figure 18; (realization of F using only NAND gates. Here eq. (2) above was used).

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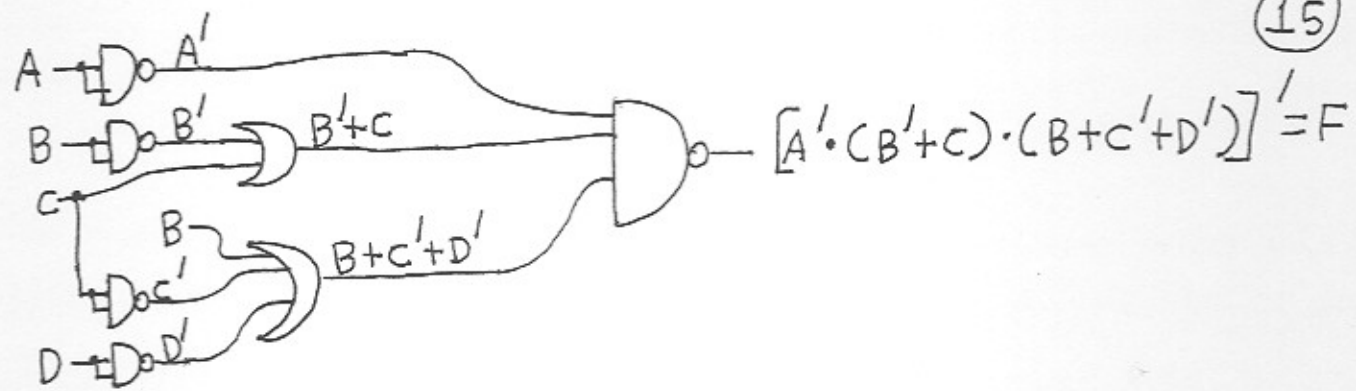


Figure 19; (realization of F using only OR and NAND gates. Here eq. (3) above was used).

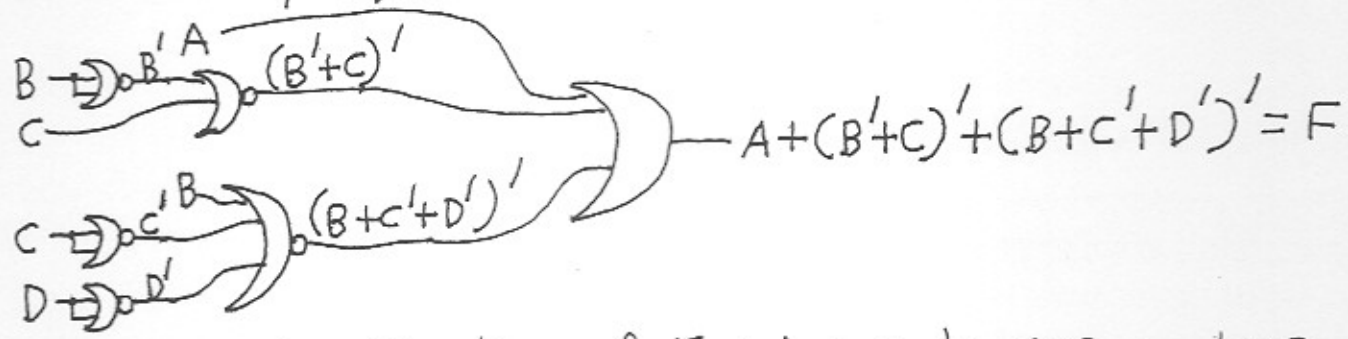


Figure 20; (realization of F using only NOR and OR gates. Here eq. (4) above was used).

I will now show you how to transform realization #4 (fig. 20) into realization #3 (fig. 19) using the graphical approach (playing with bubbles) instead of using an algebraic approach. Just see the figure below.

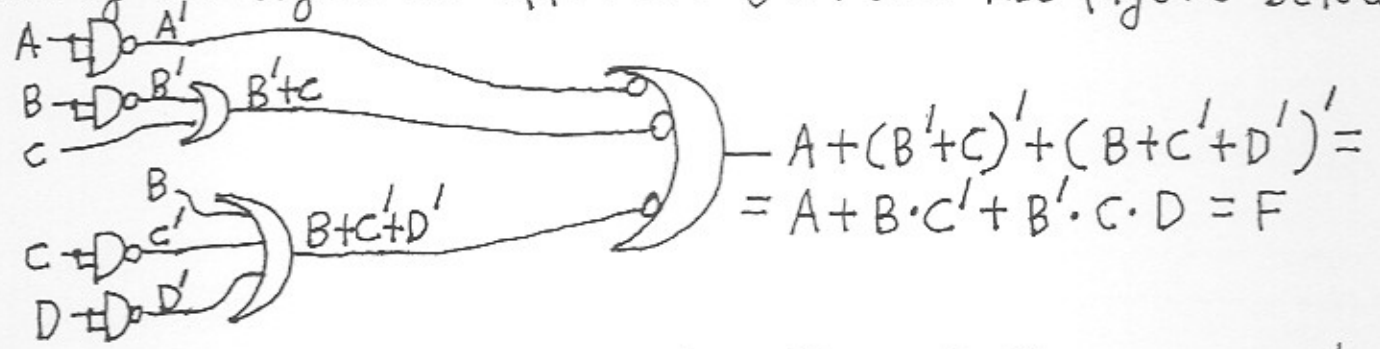


Figure 21; (another realization of F using only OR and NAND gates).

I will finally show you how to transform realization #3 (fig. 19) into realization #4 (fig. 20) using again the graphical approach (bubbles etc.) instead of using an algebraic approach. Just see the figure below.

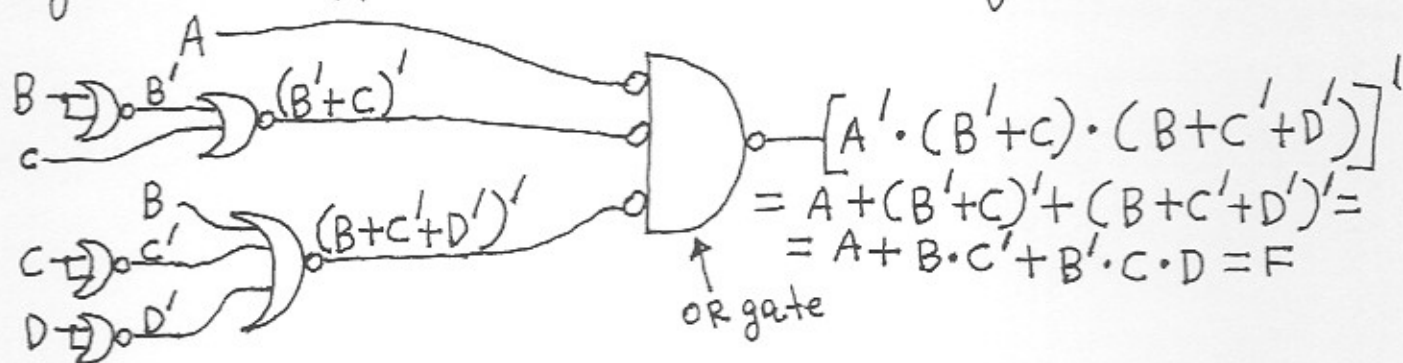


Figure 22; (another realization of F using only NOR and OR gates. The rightmost gate is an OR gate (see fig. 19 of handout #9 on last page)).

Note: Can you see how nice is the graphical approach? Enjoy the bubbles!!!

Example 6: Consider the logic function F where $F = (A+B+C) \cdot (A+B'+C') \cdot (A+C'+D)$. Realize the function F

1. Using NOT, OR and AND gates; (this is going to be realization #1).
2. Using only NOR gates; (this is going to be realization #2)
3. Using only AND and NOR gates; (this is going to be realization #3).
4. Using only NAND and AND gates; (this is going to be realization #4).

Answer: The given logic function F is already in product-of-sums form. We have:

$$F = (A+B+C) \cdot (A+B'+C') \cdot (A+C'+D) \quad (5)$$

$$= \left[\left[(A+B+C) \cdot (A+B'+C') \cdot (A+C'+D) \right]' \right]' =$$

$$= \left[(A+B+C)' + (A+B'+C')' + (A+C'+D)' \right]' \quad (6)$$

$$= (A' \cdot B' \cdot C' + A' \cdot B \cdot C + A' \cdot C \cdot D')' \quad (7)$$

$$= (A' \cdot B' \cdot C')' \cdot (A' \cdot B \cdot C)' \cdot (A' \cdot C \cdot D')' \quad (8)$$

Equation (5) above will give us realization # 1, eq. (6) will give us realization # 2, eq. (7) will give us realization # 3 and eq. (8) will give us realization # 4. The respective figures showing these four realizations follow:

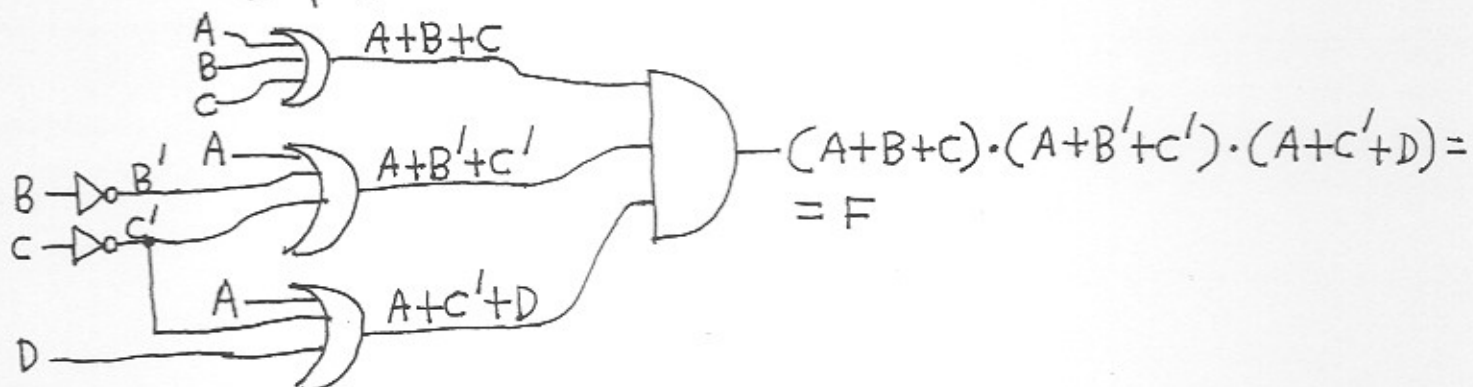


Figure 23; (realization of F using NOT, OR and AND gates. Here eq. (5) above was used).

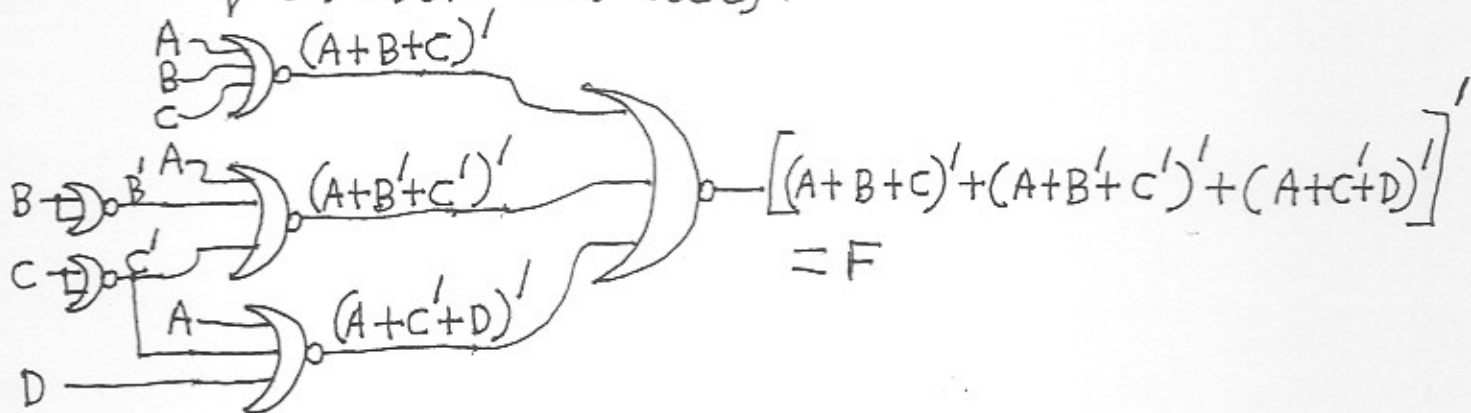


Figure 24; (realization of F using only NOR gates. Here eq. (6) above was used).

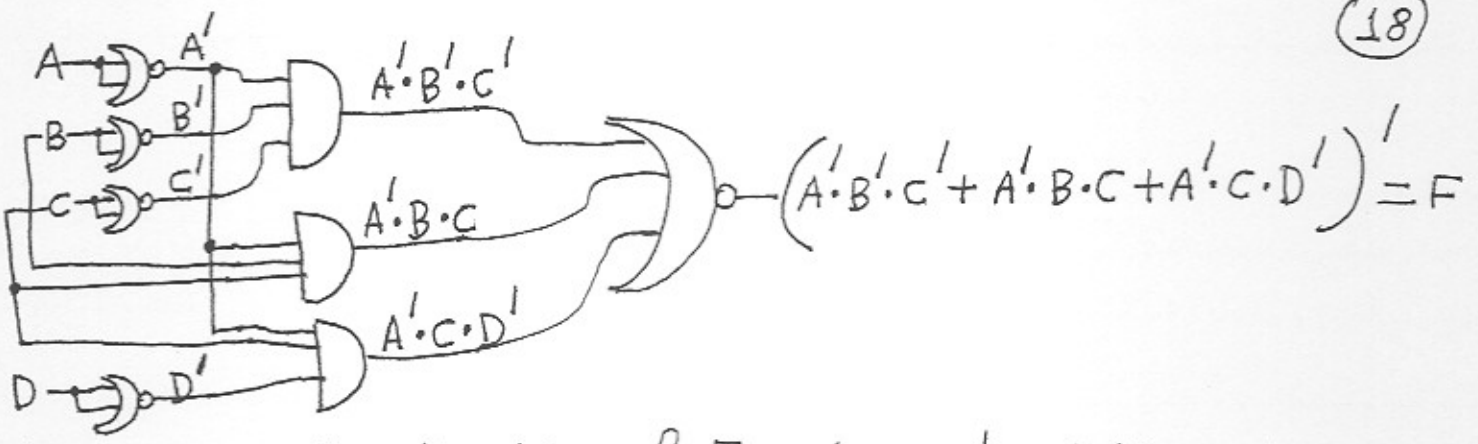


Figure 25; (realization of F using only AND and NOR gates. Here eq. (7) of previous page was used).

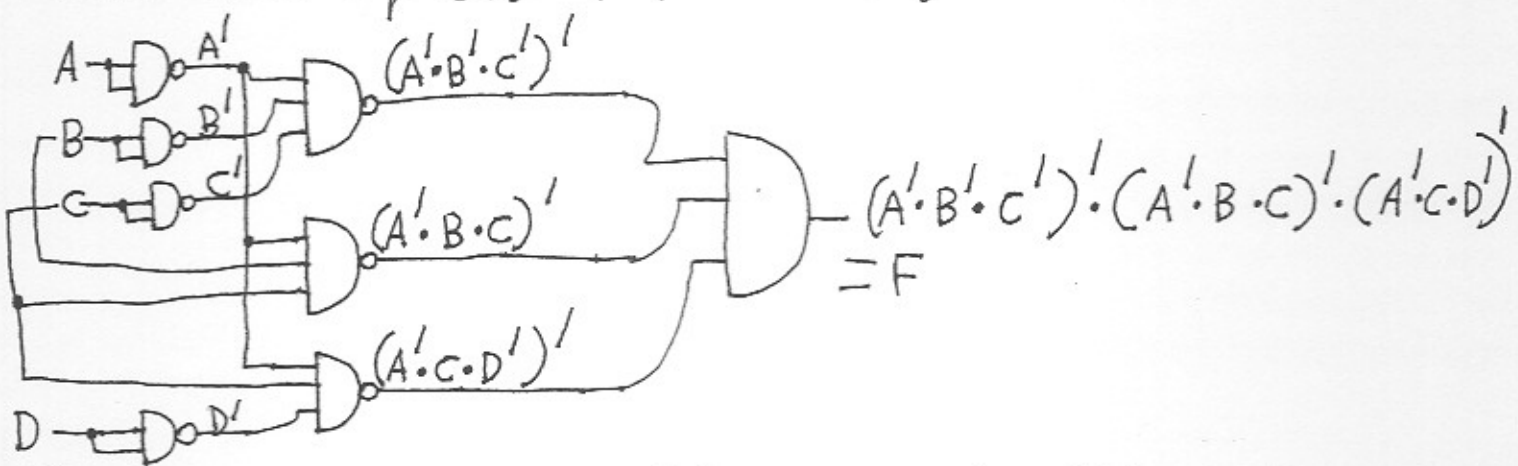


Figure 26; (realization of F using only NAND and AND gates. Here eq. (8) of previous page was used).

I will now show you how to transform realization #3 (fig. 25) into realization #4 (fig. 26), using the graphical approach (bubbles etc...) instead of using an algebraic approach. Just see the figure below.

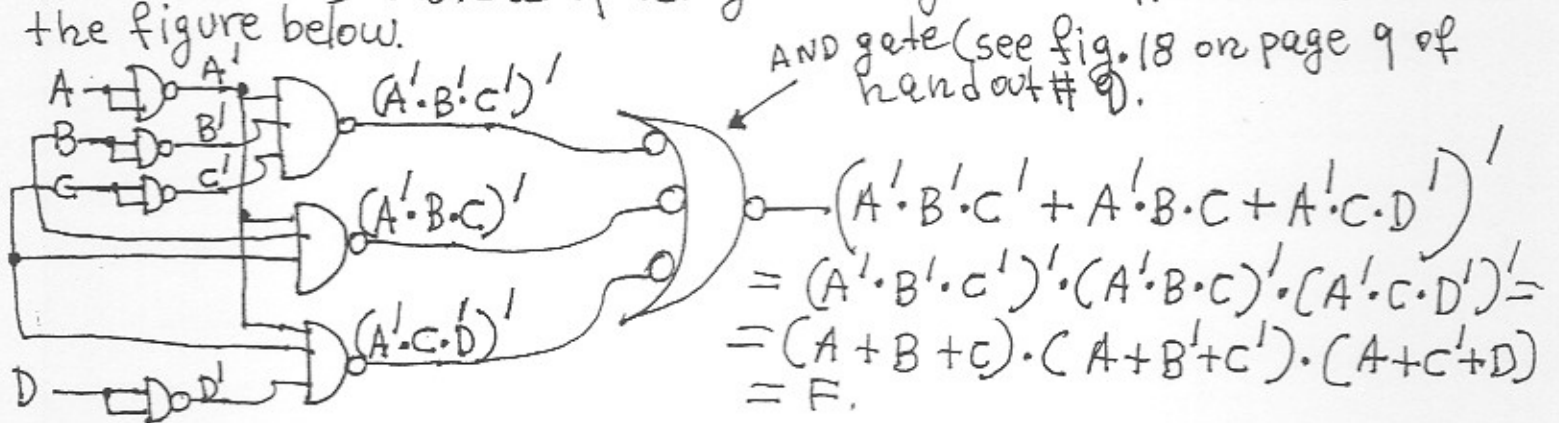


Figure 27; (another realization of F using only NAND and AND gates.)

I will finally show you how to transform realization # 4 (Fig. 26) into realization # 3 (Fig. 25), using again the graphical approach (bubbles etc...), instead of using an algebraic approach. Just see the figure below.

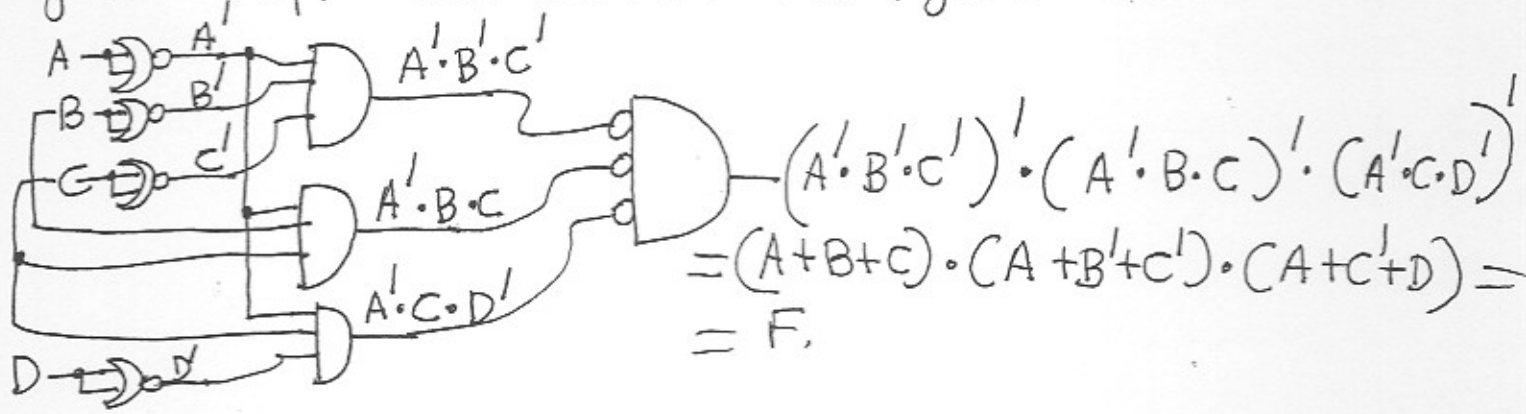


Figure 28; (another realization of F using only NOR and AND gates).

Note: By that time I ~~hope~~ hope you like the graphical approach. Enjoy the bubbles. It is lots of fun!!!!

Note: As seen from examples 5 and 6 I applied DeMorgan's theorem many times to get the different realizations. So, DeMorgan's theorem is very important.